

Production Clustering Using Dynamic Factor Analysis and K-Means++

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ABSTRACT

Clustering of daily production conditions was carried out by utilizing latent factors obtained from PT Indonesia Berlian Yasawirya's operational data. A total of 180 observations for the period January to July 2024 were analyzed using eight variables, namely the number of production, sales, stock sold, number of employees, production costs, market demand, sales speed, and number of rejected products. Dynamic Factor Analysis (DFA) was compared to Factor Analysis (FA) based on RMSE reconstruction, variance ratio, and factor interpretability. DFA produced an RMSE reconstruction of 0.1629 and a variance ratio of 0.7630, thus being chosen as the best latent representation model. The three factors formed represented market and demand dynamics, production operational pressures, and labor capacity. DFA factor scores were then standardized and grouped using K-Means with K-Means++ initialization and Agglomerative Clustering with Ward linkage in the range of the number of clusters ($k=2$) to ($k=8$). The Hopkins Statistic of 0.6077 showed a tendency for clusters at moderate levels. K-Means++ with eight clusters produced a Silhouette Score of 0.3003, a Davies–Bouldin Index of 1.0666, a Calinski–Harabasz Index of 71.3935, and a cluster balance of 0.9942. These results were better than Agglomerative Clustering in terms of the Silhouette Score, Calinski–Harabasz Index, and the balance of cluster members. In conclusion, the integrated DFA and K-Means++ pipeline successfully establishes a structured, continuous operational baseline for shop-floor monitoring rather than rigidly isolated performance categories. However, the primary limitation stems from the single-site, 180-day sample size, which constrains cross-facility generalizability and upper cluster bounds. Future research should extend this framework to multi-facility datasets and apply formal ANOVA or Kruskal–Wallis hypothesis testing against unstandardized physical metrics to externally validate descriptive latent clusters into formal efficiency tiers.



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I. INTRODUCTION

Manufacturing industries are required to maintain production efficiency and process stability amid changing market demand, fluctuating costs, and daily operational dynamics. The development of production information systems enables companies to continuously collect various operational indicators, including production volume, sales, sold stock, workforce size, production cost, market demand, selling rate, and rejected products. These data provide

valuable information for supporting data-driven decision-making. However, the large number of interacting indicators makes production conditions difficult to understand when each variable is analyzed separately.

PT Indonesia Berlian Yasawirya is one of the companies facing this challenge. The company routinely records various daily production indicators that reflect operational conditions over time. Although these data are available, the evaluation process is still dominated by separate analyses of individual indicators, preventing relationships among variables from

being observed comprehensively. Consequently, identifying operational patterns and determining production conditions becomes less effective because the process is influenced by multiple correlated variables.

The strong relationships among production indicators suggest that the variables do not operate independently but instead form a complex multivariate structure. High correlations may create information redundancy, and directly using all original variables in the analysis or clustering process may produce less representative interpretations. Therefore, a statistical approach is required to summarize the relationships among variables into a smaller number of latent factors while preserving the important information contained in the data.

One commonly used approach for this purpose is Factor Analysis (FA). FA models the covariance structure among variables to produce a lower-dimensional representation through several latent factors that explain a substantial proportion of the data variation [1], [2]. However, FA assumes that relationships among variables are static and is therefore less capable of representing patterns that change over time. To address this limitation, Dynamic Factor Analysis (DFA) extends the concept of latent factors into a state-space framework, allowing factor dynamics to be modelled according to temporal changes [3]. Zuur et al. demonstrated that DFA can estimate common trends in multivariate time-series data [3], while Ward et al. further developed this approach to improve dimension reduction and the identification of latent processes in temporally varying data [4].

Applications of DFA in various fields indicate that the method is not limited to economic or ecological data. Shoemaker et al. applied DFA to identify seasonal variation in daily physical activity [5], while Wang et al. used it to evaluate transmission-system performance based on indicators that varied across periods [6]. These studies indicate that DFA can provide a more appropriate latent representation when temporal order forms an important part of the data structure.

Although factor extraction can simplify multivariate data, the resulting factor scores do not directly describe groups of production conditions. Therefore, clustering methods are required to group observations according to similarities in their latent-factor characteristics. K-Means is widely used as a partitioning method because of its simplicity and computational efficiency, although it is sensitive to the selection of initial centroids [7], [8]. K-Means++ addresses this limitation through a more systematic centroid-initialization mechanism, thereby improving clustering stability [9]. As a comparative method, Agglomerative Clustering constructs a hierarchical structure using a bottom-up procedure without requiring initial centroids. Ward linkage merges pairs of clusters that produce the smallest increase in within-cluster variation [10].

Previous studies have shown that no single clustering algorithm consistently provides the best performance across

all data characteristics. Afrianti et al. compared K-Means with several Agglomerative Clustering methods using corrosion data and reported that the best performance depended on both the algorithm and the validation measure used [11]. Ahn and Chang applied Agglomerative Clustering to group manufacturing process models, thereby facilitating production-system management [12]. Other comparative studies have also emphasized that clustering-method selection should be based on internal validation metrics rather than determined prior to analysis [13]. Meanwhile, Zhang and Einbeck demonstrated that latent representations and clustering can be combined to improve the interpretation of patterns in multivariate data [14].

Based on these studies, factor extraction and clustering are generally discussed as separate analytical stages. Research on DFA has primarily focused on constructing latent factors, whereas clustering studies commonly use original variables or rely on a single factor-extraction method without comparing the quality of the resulting representations. Furthermore, studies that systematically compare Dynamic Factor Analysis and Factor Analysis to select the most appropriate latent representation before evaluating multiple clustering algorithms on daily production data remain limited. This selection stage is important because strong correlations among original variables may affect segmentation quality when the variables are used directly.

To address this gap, a two-stage analytical pipeline is proposed to separate latent-representation selection from clustering-algorithm selection. The first stage compares Dynamic Factor Analysis and Factor Analysis to identify the most suitable latent representation based on their ability to represent the underlying data structure. The second stage uses the selected latent representation as input for K-Means++ and Agglomerative Clustering, followed by an evaluation of segmentation quality using internal validation metrics. This approach is expected to provide a more systematic and transparent model-selection process while reducing the effects of redundancy and differences in the scales of the original variables.

The study develops an analytical framework for segmenting daily production conditions through the integration of factor-extraction and clustering methods. Its primary contribution lies in separating latent-representation selection from clustering-algorithm selection, allowing each stage to be evaluated objectively. This framework provides a more structured approach to production-data analysis and may support data-driven decision-making in manufacturing environments.

Unlike conventional static dimensionality reduction techniques such as Principal Component Analysis (PCA) or standard Factor Analysis (FA), which assume independent and identically distributed (i.i.d.) observations, daily manufacturing metrics inherently contain strong serial autocorrelation and temporal cross-lagged dependencies. Static FA collapses temporal structures into time-invariant covariance matrices, ignoring how operational shocks,

cumulative demand shifts, and production bottlenecks evolve from day $t - 1$ to day t . Dynamic Factor Analysis (DFA) overcomes this fundamental limitation by embedding latent factor trajectory estimation within a dynamic state-space framework ($\alpha_t = \alpha_{t-1} + \eta_t$). By decoupling latent dynamic extraction from subsequent partition modeling, this two-stage pipeline ensures that clustering algorithms process continuous temporal signals rather than noise-contaminated static metrics, establishing a methodological framework tailored for time-series operational telemetry.

II. METHOD

This study employed a quantitative research design based on secondary daily production data. The unit of analysis was one operational day, and no human participants were involved. The research procedure consisted of data preprocessing and statistical testing, DFA and FA modelling, factor-model evaluation, standardization of the selected DFA factor scores, comparison of K-Means++ and Agglomerative Clustering, and cluster profiling and labelling. The complete research workflow is presented in Figure 1.

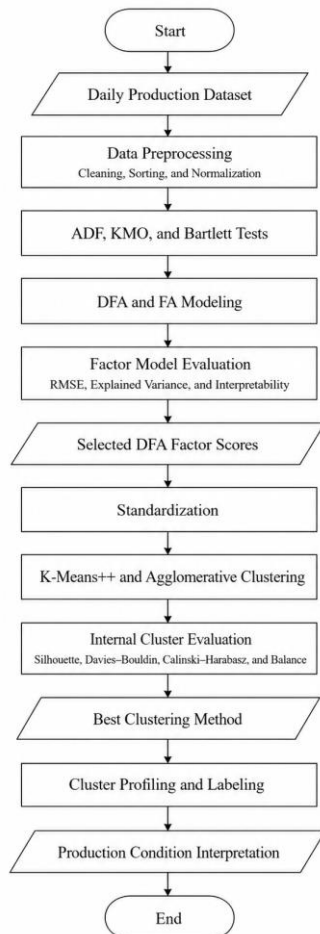


Figure 1. Research Methodology Workflow

A. Dataset and Variables

The dataset covers 180 consecutive daily observations from January to July 2024 across eight numerical variables, indexed chronologically by date without duplicate entries.

TABLE I
PRODUCTION DATASET VARIABLE

Variable	Operational Meaning
Production volume	Number of units produced per day
Sales	Daily sales value
Sold stock	Number of product units sold
Number of employees	Workforce involved in the production process
Production cost	Total daily production expenditure
Market demand	Estimated daily product demand
Selling rate	Number of units sold per hour
Rejected products	Number of products that failed the quality inspection
Production volume	Number of units produced per day

B. Preprocessing and Statistical Eligibility

Data were chronologically sorted, checked for anomalies, and normalized via Min–Max scaling to align varying measurement units. The dataset was chronologically split into training and testing sets (80:20 ratio) without shuffling. Statistical suitability for factor analysis was assessed using the Augmented Dickey–Fuller (ADF) stationarity test [15], the Kaiser–Meyer–Olkin (KMO) sampling adequacy measure [16], and Bartlett’s Test of Sphericity [17], [18].

C. Dynamic Factor Analysis and Factor Analysis

Dynamic Factor Analysis (DFA) models time-varying multivariate observations using a state-space framework [1]–[3]:

$$\alpha_t = \alpha_{t-1} + \eta_t, \quad \eta_t \sim N(0, Q) \quad (1)$$

$$y_t = Z\alpha_t + c + \varepsilon_t, \quad \varepsilon_t \sim N(0, R) \quad (2)$$

where α_t is the latent factor vector, Z is the factor-loading matrix, Q is the state noise covariance matrix, and R is the observation error covariance matrix. Candidate models (1 to 3 factors) were selected via Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) [19], [20], followed by Varimax rotation for factor interpretability [21]. As a static benchmark, Factor Analysis (FA) is defined as:

$$x = \mu + \Lambda f + e. \quad (3)$$

DFA and FA were evaluated using reconstruction RMSE across training, testing, and full datasets, variance explained ratios, and loading interpretability.

D. Preparation of DFA Factor Score

The extracted DFA factor scores (180×3 matrix) were standardized using StandardScaler to remove variance scale discrepancies prior to clustering [22]:

$$z_{it} = \frac{f_{it} - \mu_i}{\sigma_i} \quad (4)$$

Cluster ability was verified using the Hopkins Statistic (> 0.50 indicating clusterable structures relative to random distributions) [23].

E. K-Means++ and Agglomerative Clustering

K-Means++ minimizes the within-cluster sum of squares (J) using a systematic centroid initialization mechanism [9]:

$$J = \sum_{k=1}^K \sum_{x_i \in C_k} \|x_i - \mu_k\|^2 \quad (5)$$

Execution parameters were set to $n_{\text{init}}=20$, $\text{max_iter}=300$, and $\text{random state}=42$. Agglomerative Hierarchical Clustering was evaluated using Ward linkage to minimize within-cluster variance increases [10], [24]. Both algorithms were tested across cluster sizes $k = 2$ to $k = 8$.

F. Cluster Evaluation and Selection

Model evaluation combined the Silhouette Score (s_i) [25], Davies–Bouldin Index (DBI) [26], and Calinski–Harabasz Index (CHI) [27]:

$$s(i) = \frac{b(i) - a(i)}{\max(a(i), b(i))} \quad (6)$$

Cluster balance was calculated using normalized Shannon entropy ($B \in [0,1]$) [28]:

$$B = -\frac{\sum_{k=1}^K p_k \ln p_k}{\ln K}, \quad p_k = \frac{n_k}{n} \quad (7)$$

where n_k is the count in cluster k , n is total observations, and K is total clusters. Final model selection prioritized majority performance across the principal metrics (s_i , DBI, CHI) alongside cluster balance.

G. Cluster Profiling and Labelling

Cluster profiles were constructed using standardized factor mean scores:

$$Z_{c,k} = \frac{F_{c,k} - \mu_k}{\sigma_k} \quad (8)$$

Scores were categorized into high ($Z > 0.50$), low ($Z < -0.50$), or moderate ($-0.50 \leq Z \leq 0.50$) states. The relative factor contribution to centroid profiles was determined by:

$$R_{c,k} = \frac{|Z_{c,k}|}{\sum_j |Z_{c,j}|} \times 100\% \quad (9)$$

These cluster labels represent underlying production states rather than absolute efficiency. Therefore, validating them against raw operational metrics is essential before assigning final performance statuses.

III. RESULT AND DISCUSSION

A. Statistical Eligibility Result

Based on the analyses conducted, all variables had ADF p-values below 0.05. The KMO value of 0.7798 indicated that the dataset had adequate sampling suitability for factor analysis, while Bartlett's Test produced a p-value reported to be close to zero. These results supported the suitability of the dataset for factor modelling.

B. DFA Model Selection and Interpretation

Across candidate factor models, information criteria dropped consistently when moving from one to three latent factors. As Figure 2 illustrates, the three-factor model yielded the optimal fit, recording the lowest criteria values (AIC: $[-2428.046]$, BIC: $[-2306.284]$).

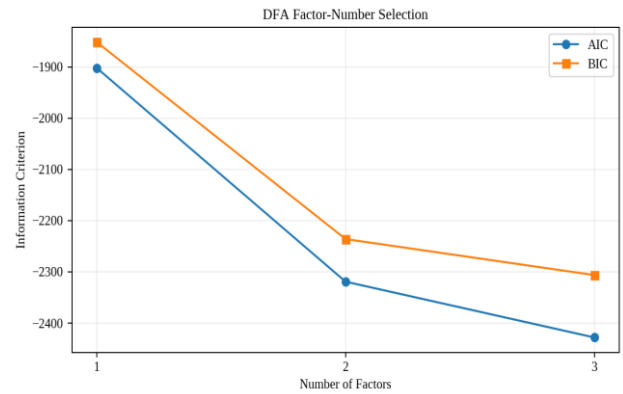


Figure 2. AIC and BIC Values for DFA Factor Selection

The Varimax-rotated factor loading matrix (Table II) reveals identical loadings for Sales and Production Cost across all factors. This symmetry stems from company policy, which sets daily production costs at a constant 25% of sales value. Consequently, Dynamic Factor Analysis (DFA) captured this deterministic relationship by assigning identical loading estimates to both variables.

TABEL II
ROTATED DFA LOADING MATRIX

Variable	Factor 1	Factor 2	Factor 3
Production volume	0.6210	-2.7189	-0.2899
Sales	2.5558	0.0841	0.1115
Sold stock	2.8433	0.1410	-0.1151

Number of employees	0.2338	-0.3646	5.2048
Production cost	2.5558	0.0841	0.1115
Market demand	3.1536	-0.1679	-0.1910
Selling rate	-0.5609	-2.1821	1.3089
Rejected products	-0.4939	-4.0403	0.1950

The three latent factors capture distinct operational dimensions. Factor 1 was primarily characterized by sales, sold stock, production cost, and market demand and was therefore labelled Market and Demand Dynamics. Factor 2 had large absolute loadings for production volume, selling rate, and rejected products and was labelled Operational Production Pressure. Factor 3 was strongly dominated by the number of employees and was additionally supported by the selling rate; therefore, it was labelled Workforce Capacity and Distribution.

Substantively, the factor loadings in Table II demonstrate intuitive physical alignments with shop-floor operations. Factor 1 (Market and Demand Dynamics) exhibits dominant positive loadings on Market Demand (3.1536), Sold Stock (2.8433), Sales (2.5558), and Production Cost (2.5558). This confirms that Factor 1 acts as a proxy for external market pull and monetary volume. Factor 2 (Operational Production Pressure) is anchored by large negative absolute loadings on Rejected Products (-4.0403), Production Volume (-2.7189), and Selling Rate (-2.1821). The magnitude of these negative loadings indicates that Factor 2 captures operational strain and quality control stress—where elevated absolute values correlate with increased defective yield and throughput bottlenecks. Factor 3 (Workforce Capacity and Distribution) is heavily dominated by Number of Employees (5.2048) alongside Selling Rate (1.3089), representing available labor deployment constraints. The structural consistency of these dynamic loadings confirms that DFA isolates meaningful operational drivers rather than arbitrary variance components.

Unlike static Factor Analysis (FA), DFA loadings function as coefficient parameters within matrix $\mathbf{Z}$ rather than strict correlation bounds ($[-1, 1]$).

C. DFA vs FA Performance Comparison

Across every partition, DFA easily outperformed static FA. Looking at the complete dataset, DFA dropped the reconstruction RMSE to 0.1629 (compared to 0.5067 under FA) while boosting the explained variance ratio to 0.7630 (versus 0.3697 for FA), a contrast highlighted in Figure 3.

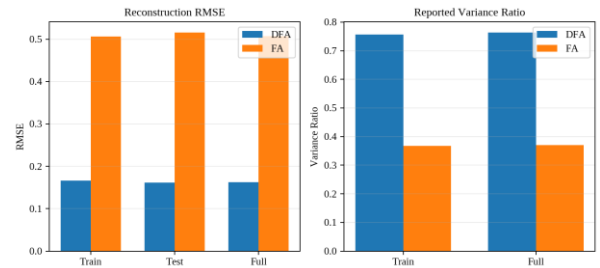


Figure 3. Reconstruction and Explained-Variance Comparison Between DFA and FA

Cross-model comparison via Pearson correlation revealed high convergence between DFA Factor 1 and FA Factor 1 ($r = 0.88$, $p < 0.001$). DFA Factor 2 correlated moderately and inversely with FA Factor 3 ($r = -0.54$, $p < 0.001$), while DFA Factor 3 aligned moderately with FA Factor 2 ($r = 0.61$, $p < 0.001$). While both approaches identified related latent dimensions, DFA's state-space structure effectively captured temporal dynamics that static FA failed to resolve [3], [4]. Consequently, only DFA factor scores were selected for the clustering pipeline.

D. Factor-Score Properties and Cluster ability

Standardized DFA factor scores yielded a Hopkins Statistic of 0.6077 ± 0.0467 , confirming a moderate clustering tendency relative to a uniform random baseline ($H = 0.50$). This outcome shows that daily plant operations undergo smooth, continuous transitions between states rather than forming rigidly separated groups.

E. Clustering Evaluation and Algorithm Selection

The standardized DFA factor scores yielded a Hopkins Statistic of 0.6077 ± 0.0467 , confirming moderate clusterability significantly above a random uniform baseline ($H = 0.50$). In manufacturing systems, a moderate Hopkins index reflects continuous transitions across operational states rather than discrete, sharply segregated clusters. As daily plant operations fluctuate dynamically due to shifting demand and machine throughput, boundaries between consecutive daily states naturally blur. This continuous manifold explains the modest peak Silhouette Score of 0.3003 achieved by K-Means++ at $k = 8$ (Figure 4). While an absolute Silhouette Score of 0.3003 indicates moderate cluster boundary overlap, $k = 8$ consistently prioritized structural balance across multiple metrics, simultaneously optimizing the Davies–Bouldin Index (1.0666), Calinski–Harabasz Index (71.3935), and normalized-entropy balance (0.9942).

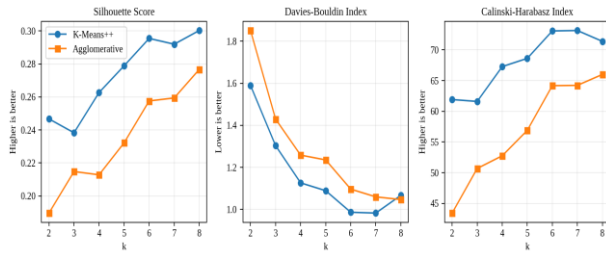


Figure 4. Internal Validation Metrics Across Candidate Cluster Numbers

Figure 5 and Figure 6 summarize the performance and cluster size distributions comparing K-Means++ and Agglomerative Clustering at $k=8$. K-Means++ demonstrated superior Silhouette Score (0.3003 vs 0.2767), CHI (71.3935 vs 66.0088), and normalized-entropy balance (0.9942 vs 0.9787), whereas Agglomerative Clustering yielded a slightly better DBI (1.0466 vs 1.066). K-Means++ maintained a balanced membership distribution (15–26 observations per cluster), avoiding the oversized cluster $n = 39$ formed by Ward linkage.

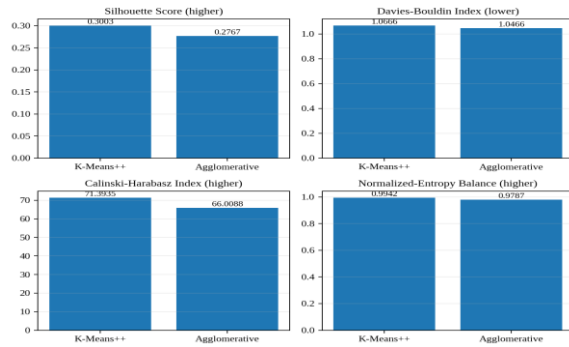


Figure 5. Final Clustering Metrics Using Their Original Scales

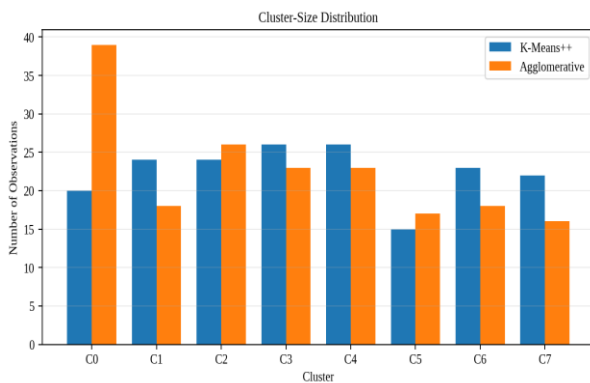


Figure 6. Cluster-Size Distribution of the Compared Methods

F. Cluster Profiling and Master Profiles

For the optimal $k = 8$ K-Means++ model, Table III consolidates the standardized cluster factor profiles ($Z_{c,k}$) and relative factor contributions ($R_{c,k}$), with Figure 7 providing the corresponding visual mapping.

TABEL III
MASTER PROFILE SUMMARY OF SELECTED K-MEANS++ CLUSTERS (K=8)

Cluster	n	Z-F1	Z-F2	Z-F3	R-F1 (%)	R-F2 (%)	R-F3 (%)	Descriptive Latent Profile
C0	20	0.84	-0.31	-1.33	33.9%	12.6%	53.5%	High F1 – Low F3
C1	24	-1.08	-0.69	0.68	44.1%	28.2%	27.7%	Low F1 – High F3
C2	24	-0.69	0.89	0.98	26.8%	34.9%	38.3%	High F2 – High F3
C3	26	0.86	0.31	-0.24	60.9%	22.0%	17.2%	High F1
C4	26	0.67	-0.83	0.87	28.2%	35.1%	36.7%	High F1 – Low F2 – High F3
C5	15	1.09	1.10	0.94	34.8%	35.2%	30.0%	All-Factors-High
C6	23	-0.65	1.09	-0.99	23.7%	39.9%	36.3%	High F2 – Low F1/F3
C7	22	-0.70	-1.20	-0.95	24.4%	42.2%	33.4%	All-Factors-Low

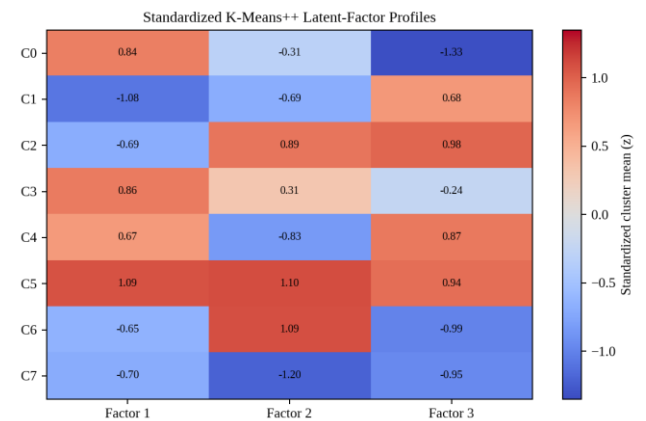


Figure 7. Standardized Latent-Factor Profiles of the Selected K-Means++ Clusters

Relative factor shares ($R_{c,k}$) quantify how much each cluster centroid deviates within the latent space. Factor 1 (Market and Demand Dynamics) heavily dominates cluster C3, whereas cluster C0 stems mainly from Factor 3 (Workforce Capacity and Distribution). Meanwhile, C5 reflects high operational intensity across all three dimensions, standing in sharp contrast to C7's low baseline activity across the board.

Table III presents the relative shares of each factor's absolute centroid deviation. For example, Cluster 3 (C3) had an R-F1 value of 60.9%, whereas Cluster 0 had an R-F3 value of 53.5%. These percentages were not factor loadings, causal weights, or indicators of clustering quality. They only represented the proportion of each centroid's absolute deviation from the global mean within the latent-factor space.

Cluster 5 (C5) was above the overall mean for all three factors and was therefore described as an **All-Factors-High Latent Profile**. Cluster 7 (C7) was below the mean for all three factors and was described as an **All-Factors-Low Latent Profile**. Cluster 3 (C3) was primarily characterized by a high Factor 1 score, whereas Cluster 6 (C6) had a high Factor 2 score combined with low Factor 1 and Factor 3 scores. Factor codes were intentionally retained in these descriptions to avoid implying operational performance that had not yet been empirically validated.

Cluster 0 (C0) exhibited a high Factor 1 score and a low Factor 3 score, whereas Cluster 1 (C1) exhibited a low Factor 1 score and a high Factor 3 score. Cluster 2 (C2) showed high Factor 2 and Factor 3 scores, while Cluster 4 (C4) showed high Factor 1 and Factor 3 scores together with a low Factor 2 score. These labels only described the relative configurations of the factor scores.

Because factor signs are not uniquely determined, the final operational interpretation should refer to both the loading patterns and the average original-variable values within each cluster.

TABEL IV
EXTERNAL VALIDATION PLAN FOR OPERATIONAL LABELS

External Indicator	Cluster Statistic	Validation Question	Possible Operational Use
Production volume	Mean/median by cluster	Do clusters differ in output level?	Output-capacity categorization
Market demand and sales	Mean and fulfillment ratio	Are market-driven states commercially distinct?	Demand-response planning
Rejected products	Reject rate per cluster	Does operational pressure coincide with quality loss?	Quality-control prioritization
Production cost	Cost per unit	Do clusters differ in resource efficiency?	Cost-control intervention
Number of employees	Output per employee	Does workforce state reflect productivity?	Workforce allocation
Production efficiency	Mean and distribution	Are latent states associated with performance?	Performance-based label validation

To develop the labels from latent-state descriptions into operational-performance labels, cluster assignments should be added to the final dataset, after which the mean production volume, demand, sales, cost, rejected products, workforce

size, and production efficiency should be calculated for each cluster.

ANOVA or the Kruskal–Wallis test may be used in future external-validation analyses to determine whether the operational indicators differ significantly among clusters. Without such validation, labels such as *high performance* or *inefficient* should not be used because they would not yet be supported by external operational evidence.

IV. CONCLUSION

This study established a two-stage analytical pipeline for segmenting daily manufacturing conditions by integrating Dynamic Factor Analysis (DFA) and K-Means++ clustering. The empirical results demonstrate that state-space temporal modeling via DFA substantially improves latent representation quality over static FA, reducing reconstruction RMSE from 0.5067 to 0.1629 and increasing explained variance from 0.3697 to 0.7630 across 180 daily observations. In the clustering stage, K-Means++ at $k = 8$ provided a balanced partitioning of operational states. Given the moderate cluster separation (Silhouette Score = 0.3003; Hopkins Statistic = 0.6077), the pipeline should be interpreted as generating a structured, continuous operational baseline rather than rigidly isolated performance categories.

The primary limitations of this study stem from the single-site, 180-day sample size, which constrains cross-facility generalizability and risks sensitivity to localized data variations. Furthermore, selecting $k = 8$ at the upper boundary of the evaluated search range ($k = 2$ to $k = 8$) suggests that higher cluster resolutions could exist. Future research should extend this framework to multi-year, multi-facility datasets, incorporate bootstrap sensitivity testing to evaluate cluster centroid stability, and conduct formal ANOVA or Kruskal–Wallis hypothesis testing against unstandardized financial metrics to externally validate descriptive latent clusters as formal efficiency tiers.

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