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Numerical Study of Secant Method for Finding the Optimum RF Coil Length in MRI Systems Using Python

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ABSTRACT

Determining the coil length *l* to achieve the desired target inductance L_{target} involves nonlinear and transcendental RF coil design equations. The ideal solenoid coil model does not align with real-world conditions, therefore, it is corrected using the Wheeler and Nagaoka models. From a mathematical perspective, these two models are nonlinear, making it highly difficult to determine the coil length analytically. This study presents the application of the Secant method to solve the coil length optimization across three models: Ideal Solenoid, Wheeler, and Nagaoka. By applying the Secant method with a case study involving a target inductance L_{target} = 10 µH, radius r = 5 cm, and number of coil turns N = 15, an optimum length of 17.71 cm was obtained with an error tolerance of 10^{-6}. The convergence rates of the three models were evaluated to obtain error value data at each iteration. Based on the relationship between the convergence rate and iteration steps, the Wheeler and Nagaoka models yielded identical data at every iteration. The Secant method proved effective in solving nonlinear function root-finding cases, demonstrating a logarithmic convergence rate. This study is expected to provide a reliable computational framework for medical device engineers to ensure manufacturing accuracy in the design of RF coils in MRI systems.

I. Introduction

Magnetic Resonance Imaging (MRI) technology, as depicted in Figure 1, is one of the most powerful non-invasive diagnostic tools in modern medicine [1]. The heart of the system is a Radio Frequency (RF) coil that generates an oscillating magnetic field B_1 and detects signals from the patient's body [2]. For the system to operate with maximum sensitivity, the RF coil must resonate precisely at the Larmor frequency, which is determined by the strength of the main magnetic field B_0 parallel to the z-axis [3]. For example, in a 1.5 Tesla system, the coil must operate at a frequency of 63.87 MHz [4]. The RF coil generates an oscillating magnetic field B_1 perpendicular to the static main magnetic field B_0 [2]. For a cylindrical scanner like Figure 2, the magnetic field B_0 is directed in the z direction along the scanner bore, while the magnetic field B_1 is in the transverse plane (xy) [5]. If the oscillations of B_1 closely match the natural frequency near the Larmor frequency (the natural frequency of the "vibration" (precession) of protons in the main magnetic field B_0, energy will be absorbed into the protons, resonance causing them to start moving in unison (phase coherence) [6]. As a result, a torque force is generated that deflects the direction of the protons' spin axis from the direction parallel to the main field B_0 (Z-axis) towards the plane perpendicular to it (XY-plane). The longer (several milliseconds) the RF pulse is applied, the larger the deflection angle (e.g., 90° or 180°) [6].

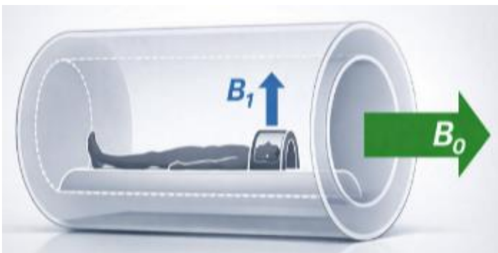


Figure 1. MRI System [7]

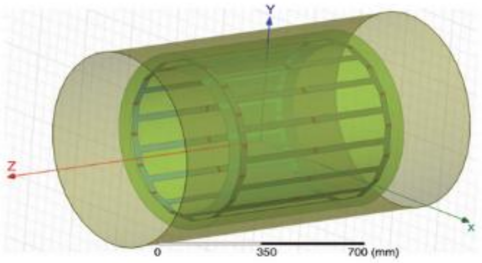


Figure 2. RF Coil Model in MRI [8]

The resonance of an RF coil circuit is highly dependent on the inductance value $\boldsymbol{L}$ of the coil geometry [9]. In the fabrication stage, hardware engineers are often faced with the problem of determining the physical geometry parameters—particularly the correct solenoid length l to achieve the target inductance value specified in the tuning circuit design [10]. Traditionally, inductance calculations often use an idealized approach, but in MRI coils with short aspect ratios, the ideal model fails to provide accurate results due to the effects of flux leakage at the coil ends [11], [3]. Mathematical problems arise when more accurate models are used, such as the Wheeler model or the Nagaoka model equipped with wire thickness corrections [8]. The Wheeler and Nagaoka models are nonlinear and often transcendental, where the correct solenoid length l appears both as the denominator and in the geometric correction function. This makes analytical isolation of the variable l very difficult or even impossible [10].

This study aims to obtain approximate solutions to the roots of nonlinear equations using the Secant method. Based on research [12], [13], [14], [15], [16], [17], [18], [19], [20] the Secant method is preferred because of its efficiency in finding the roots of a nonlinear function without the need to calculate the first derivative, which can be difficult to determine. The Secant method proves particularly beneficial when dealing with complex empirical models [21], [14], [22]. This paper delves into a detailed comparison of three mathematical models of RF coils: the *Ideal solenoid* model, the *Wheeler* model and the *Nagaoka* model. These three models are solved using the Secant method to ascertain the value of the solenoid length, a critical physical variable in experimental design. Through this study, it is hoped that a rigid and precise MRI hardware development design for the future can be obtained [1], [9].

II. METHOD

This study employs a numerical computational approach based on the Secant method to find solutions to mathematical models (nonlinear functions in solenoid design). The numerical computation of the Secant algorithm is implemented using the Python 3 programming language. Three solenoid models will be solved numerically using the Secant method. This method is used to determine the optimal coil length which is equivalent to finding the roots of the nonlinear functions of the three solenoid models. The convergence rate in achieving the coil length value (root of the nonlinear function) for the three solenoid models will be compared to evaluate the effectiveness of the Secant method in solving nonlinear mathematical model problems.

A. Ideal Solenoid Model

The inductance equation for a single layer solenoid assumes an ideal solenoid with no flux leakage, expressed by equation (1) [4].

$$L = \frac{\mu_0 . N^2 . (\pi r^2)}{l}. \tag{1}$$

Where L : coil inductance ; μ_0: space permeability, $4\pi.10^{-7}$ (H/m) ; N : number of coil turns ; r : coil radius ; and l : length of the coil

For high accuracy on short coils (often used in MRI), a modification is used that includes correction factors for coil length and radius as stated by equation (2) [18].

$$f(l,r) = \frac{\mu_0 . N^2 . \pi . r^2}{l+0.9r} - L_{target} = 0. \tag{2}$$

B. WheelerModel

The Wheeler model is often used for single-layer solenoids because it has an accuracy of about 1% for a given dimension ratio [1]. The Wheeler solenoid equation is expressed in equation (3)

$$L = \frac{r^2 . N^2}{9r+10l} \tag{3}$$

To determine the value of l, equation (3) is converted into a root function as in equation (4),

$$f(l,r) = \frac{r^2 . N^2}{9r+10l} - L_{target} = 0 \tag{4}$$

C. Nagaoka Model

The Nagaoka model is the standard model for MRI coil design because it includes a correction factor K that compensates for the loss of magnetic flux at the solenoid tip as expressed by equation (5) [10]

$$L = K . \frac{\mu_0 \, N^2 . \pi r^2}{l} \tag{5}$$

The Nagaoka correction factor K itself is a function of the diameter to length ratio *(2r/l)*, as stated by equation (6) [10]

$$K = \frac{1}{1+0.45\left(\frac{2r}{l}\right)-0.005\left(\frac{2r}{l}\right)^2} \tag{6}$$

From the mathematical form of equation (6), the variable l appears in the main denominator of equation (5) as well as in the function *K(l)*. This makes the equation transcendental. To obtain the value of l, equation (6) is transformed into a root function as in equation (7)

$$f(l,r) = K(l) . \frac{\mu_0 \, N^2 . \pi r^2}{l} - L_{target} = 0 \tag{7}$$

D. Secant method

The Secant method was chosen as the numerical method for optimizing the root value search [14], [13], [19], because it does not require the calculation of the first derivative of *f'(l)*, as does the Newton-Raphson method which requires information on the first derivative of the function. This is advantageous if the inductance function becomes more complex in the future [9]. The iteration formula for the Secant method to determine the optimum coil length is stated in equation (8) [22],

$$l_{n+1} = l_0 - f(l_n)\frac{l_n - l_{n-1}}{f(l_n) - f(l_{n-1})} \quad (8)$$

E. Flowchart

The flowchart of the implementation of the Secant method algorithm in the case of searching for the optimum value of the coil length l is depicted in Figure 3.

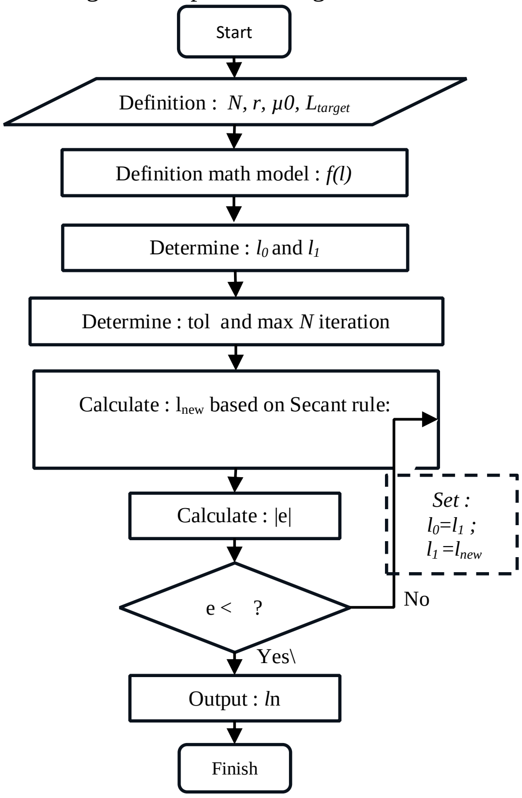


Figure 3. The Flowchart of finding the optimum coil length l

F. Implementation of Secant Algorithm

The implementation of the Secant algorithm for searching the value of the coil length l which is the root of the nonlinear equation in the form of equations (2), (4) and (7) with a numerical tolerance ε, is described as follows:

1) Define the constant values: N, r, $\mu0$, and L_{target}
2) Define the function *f(l)* according to the equation model (ideal solenoid/Wheeler/Nagaoka).
3) Determine two initial guesses of the coil length, l_0 and l_1.
4) Determine the tolerance ε (e.g. 10^{-6}) and the desired maximum number of iterations.
5) Run the Secant method iteration:
    a. Calculate the new l using the Secant formula equation (8);
    b. Calculate the realtive error e =$|l_{n+1} - l_n| / l_n$
6) If error e < tol ε then l_n is the solution; otherwise (error e > tol ε) then update $l_{n-1} = l_n$ and $l_n = l_{n+1}$ Repeat step 5.
7) The iteration is complete and displays the coil length results l_n.

III. RESULTS AND DISCUSSION

In this study, the input parameters of the RF coil design case study for the three coil models (ideal solenoid model, *Wheeler* model and *Nagaoka* Model) are determined uniformly; L_{target} = 10 μH =1x10^{-5}H, N = 15 turns, and r = 5 cm = 0.05 meters. In Python scripts, the function definition or coil model is written at the beginning after the parameter definition to facilitate changing the mathematical model without changing the iteration structure of the Secant method [23], [24]. For example, the Wheeler model, the function definition is written as follows.

```
def f_wheeler(l,N,r,L_t):
#    Wheeler Model
    r_in, l_in = r/0.0254, l/0.0254
    L_uH =(r_in**2*N**2)/(9*r_in+10*l_in)
      return (L_uH * 1e-6) - L_t
```

While the Nagaoka model, the function definition is written in the Python script as follows.

```
def f_nagaoka(l, N, r, L_target):
 # Faktor koreksi K (pendekatan)
 K=1/(1+0.45*(2*r/l)-0.005*(2*r/l)**2)
 #  Nagaoka Model
L_calc=K*(MU_0*N**2*np.pi*r**2)/l
  return L_calc - L_target
```

The Python script snippet for implementing the Secant method algorithm to find the root value of a function is written as follows.

```
    def  secant_method(l0,  l1,  N,  r,  L_target, tol=1e-10, max_iter=100):
        l_prev = l0
        l_curr = l1
        print(f"{'Iter':<5} | {'Length l (m)':<15} | {'Error':<15}")
        print("-" * 40)
        for i in range(max_iter):
            f_prev = f(l_prev, N, r, L_target)
            f_curr = f(l_curr, N, r, L_target)
            if f_curr - f_prev == 0:
                break
            # Formulation of Secant iterasion :
    l_new=l_curr-f_curr*(l_curr-l_prev)/(f_curr-f_prev)
            error = abs(l_next - l_curr)
            print(f"{i+1:<5}  |  {l_next:<15.6f}  | {error:<15.2e}")
            if error < tol:
                return l_new
            l_prev = l_curr
            l_curr = l_new
      return l_curr
```

In this case study, the tolerance value ε is set at 10^{-6}. When the error e has been achieved (e < 10-6), the coil length value l is obtained. The graph of the coil inductance as transcendent function $L_{(l)}$ - L_{target} against the coil length l of the ideal solenoid model based on the Secant method to obtain the value of the coil length l is depicted in Figure 4. From the computation results by trying the initial values l_0 = 0.01 meters and l_1 = 0.10 meters, the root value of the function or the coil length value is 0.177066 meters as in Figure 5.

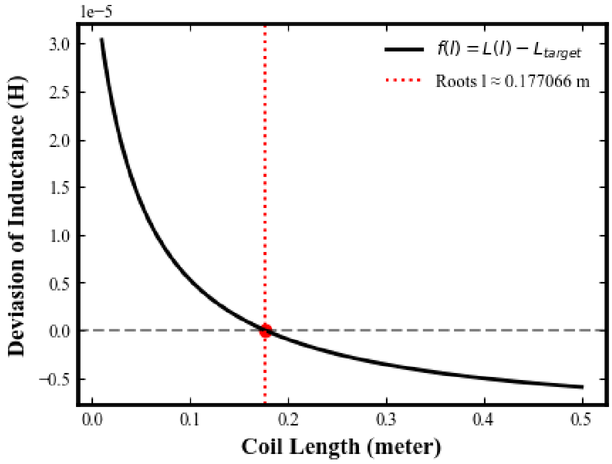
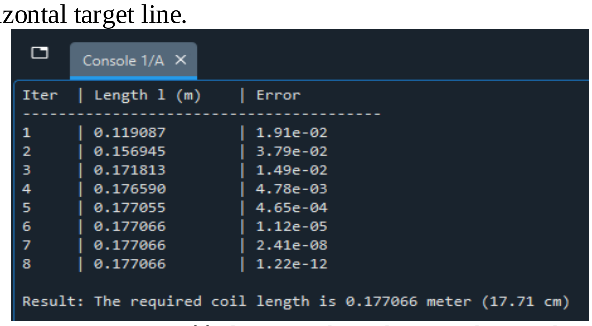


Figure 4. The Secant methods for roots finding of nonlinier function

The graph in Figure 4 visualizes the application of the Secant method to find the root of a nonlinear equation in determining the coil length precisely. The vertical axis represents the inductance deviation function, $f(l) = L(l) - L_{target}$ where the main objective of this optimization is to find the value of coil length l that makes the deviation zero. The red dot on the graph indicates the location of the root of the function at 0.177066 meters, which represents the intersection point between the inductance characteristic curve and the horizontal target line.



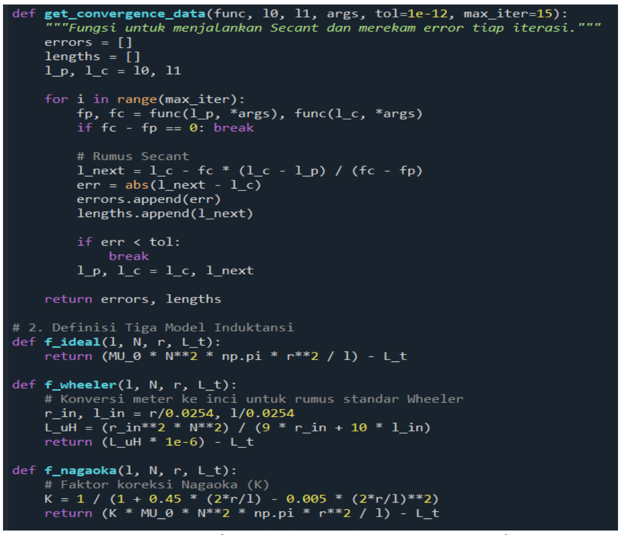
```
Console 1/A

Iter | Length l (m)    | Error
---------------------------------------
1    | 0.119087        | 1.91e-02
2    | 0.156945        | 3.79e-02
3    | 0.171813        | 1.49e-02
4    | 0.176590        | 4.78e-03
5    | 0.177055        | 4.65e-04
6    | 0.177066        | 1.12e-05
7    | 0.177066        | 2.41e-08
8    | 0.177066        | 1.22e-12

Result: The required coil length is 0.177066 meter (17.71 cm)
```

Figure 5. Iteration of finding root value and numerical error value

The output results of the numerical simulation of the Secant method in the Python programming environment are shown in Figure 5. From the eight iteration stages of the Secant method, it can be seen that the convergence process of the root value from the initial estimate has been achieved reaching a precision figure of 0.177066 meters or approximately 17.71 cm. The decrease in the error value is very significant on the order of 1.22×10^{-12} indicating that the Secant method algorithm works very accurately and has succeeded in finding the optimum solution for the coil design length value l.



```python
def get_convergence_data(func, l0, l1, args, tol=1e-12, max_iter=15):
    """Fungsi untuk menjalankan Secant dan merekam error tiap iterasi."""
    errors = []
    lengths = []
    l_p, l_c = l0, l1

    for i in range(max_iter):
        fp, fc = func(l_p, *args), func(l_c, *args)
        if fc - fp == 0: break

        # Rumus Secant
        l_next = l_c - fc * (l_c - l_p) / (fc - fp)
        err = abs(l_next - l_c)
        errors.append(err)
        lengths.append(l_next)

        if err < tol:
            break
        l_p, l_c = l_c, l_next

    return errors, lengths

# 2. Definisi Tiga Model Induktansi
def f_ideal(l, N, r, L_t):
    return (MU_0 * N**2 * np.pi * r**2 / l) - L_t

def f_wheeler(l, N, r, L_t):
    # Konversi meter ke inci untuk rumus standar Wheeler
    r_in, l_in = r/0.0254, l/0.0254
    L_uH = (r_in**2 * N**2) / (9 * r_in + 10 * l_in)
    return (L_uH * 1e-6) - L_t

def f_nagaoka(l, N, r, L_t):
    # Faktor koreksi Nagaoka (K)
    K = 1 / (1 + 0.45 * (2*r/l) - 0.005 * (2*r/l)**2)
    return (K * MU_0 * N**2 * np.pi * r**2 / l) - L_t
```

Figure 6. Python script for calculating the convergence of three RF coil models

The convergence rate of the three methods was determined by creating separate Python scripts to facilitate comparison of the speed at which the three methods obtain the root value of the function, namely the length of the RF coil. An excerpt of the Python script for calculating the convergence rate is depicted in Figure 6. Meanwhile, Table I shows the results of calculating the convergence rates of the three models.

TABLE I
ABSOLUTE ERROR VALUE

Iteration	Ideal Solenoid	Wheeler model	Nagaoka model
1	2.13×10^{-2}	9.34×10^{-2}	9.34×10^{-2}
2	1.00×10^{-1}	1.00×10^{-1}	1.00×10^{-1}
3	3.22×10^{-2}	8.30×10^{-2}	8.30×10^{-2}
4	7.25×10^{-3}	1.00×10^{-2}	1.00×10^{-2}
5	8.12×10^{-4}	6.20×10^{-3}	6.20×10^{-3}
6	1.07×10^{-5}	1.08×10^{-4}	1.11×10^{-4}
7	4.76×10^{-7}	1.08×10^{-6}	1.08×10^{-6}
8	8.52×10^{-11}	1.00×10^{-9}	1.00×10^{-9}
9	1.00×10^{-17}	5.01×10^{-14}	5.01×10^{-14}

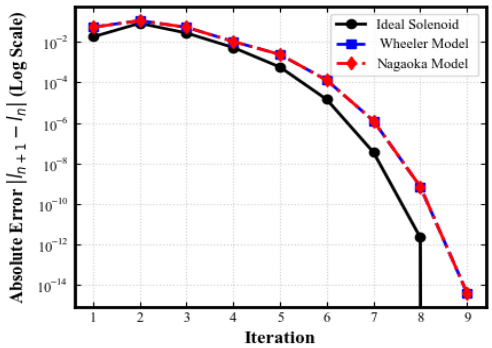


Figure 7. The convergence graphs of the three models coil

As seen in Figure 7, the convergence graphs of the three models show a very sharp decrease in error (forming a descending line on a logarithmic scale). The absolute error value scaled logarithmically, is used as a parameter to indicate how quickly the iteration reaches convergence. This shows that the Secant method has a very fast (nonlinear) convergence rate. In this case, it takes less than 10 iterations to obtain the precise coil length. Based on the convergence graph profiles of Figure 7, it appears that the Nagaoka and Wheeler models coincide and are similar to the ideal solenoid curve.This indicates that although the Wheeler and Nagaoka equations are much more mathematically complex because they involve the variable l in the nonlinear correction factor K), the Secant method does not experience a significant slowdown in convergence.

IV. CONCLUSION

This study successfully applies the Secant method as a numerical method to solve the problem of finding the roots of nonlinear functions that arise in MRI coil design. In RF coil design, coil design precision is paramount. If the coil length is miscalculated, the circuit's resonant frequency can shift from the required Larmor frequency. This study successfully modeled three RF coil designs: namely the ideal solenoid model, the Wheeler model, and the Nagaoka model. Based on the three models and a case study with uniformly determined input parameters L_{target} = 10 μH, N = 15 turns, and r = 5 cm = 0.05 m, the root value of the function for coil length was obtained as 0.177066 meters (17.71 cm) with a tolerance of 10^{-6}. Based on the three models of coils, the Secant method has been proven effective in solving cases where the root of a nonlinear function of an RF coil model converges logarithmically.

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