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Enhanced Multi-Objective Green Vehicle Routing with a New Fuzzy Speed-Driven Fuel Consumption Model

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ABSTRACT

Today, decision-makers begun to prioritize the concept of green logistics, which is based on strategies aimed to promote more environmentally sustainable practices during vehicle routing. Among key factors influencing fuel consumption in such problems, vehicle speed plays a crucial role. This article adapts the Comprehensive Modal Emission Model (CMEM) for fuel consumption by treating vehicle speed as a fuzzy variable. This enhanced version, referred as Fuzzy-CMEM, enables the formulation of a more realistic fuzzy multi-objective Green Vehicle Routing Problem (GVRP). The proposed methodology follows four main steps. First, we formulate the problem considering the vehicle speed as a fuzzy variable. Second the initial fuzzy problem is defuzzified using the interval approximation approach. Third, a sequential approach is adopted where the sweep heuristic is used to construct feasible routes, and the BicriterionAnt metaheuristic is employed to generate optimal Pareto-front solutions of the resulting deterministic problem. Finally, a numerical simulation is addressed, followed by a comparative analysis of results and discussion.

I. INTRODUCTION

Vehicle speed plays a crucial role in fuel consumption and pollutant emissions in the transportation systems. In the context of the Green Vehicle Routing Problem (GVRP), incorporating speed as an influential variable enables a more realistic modeling of both energy and environmental costs.

Since the seminal work of Dantzig and Ramser [1] on the Vehicle Routing Problem (VRP), research in vehicle routing optimization has significantly evolved, particularly by integrating environmental aspects. This evolution has led to the formulation of the Green Vehicle Routing Problem (GVRP) [2], which aims to minimize not only economic cost or logistical costs, but also greenhouse gas emissions and energy consumption. This problem aligns with the global shift toward ecological transportation, where the search for sustainable routing solutions has become a strategic priority, as reported by the International Energy Agency (IEA) [3].

The studies by Bektaş and Laporte [4], as well Kara et al. [5], have emphasized the need to consider both economic and environmental criteria simultaneously in routing models, giving rise to multi-objective formulations of the GVRP. In order to accurately assess the environmental impact of each route, several fuel consumption models have been developed. In addition to Zhang's model [6], these models can broadly be categorized into two main types: macroscopic and microscopic approaches [7,8]. Macroscopic models, such as those reviewed by Hickman [9] or employed in early emission minimization studies like the one of Figliozzi [10], are generally based on aggregate functions that relate fuel consumption to average speed, travel distance, or vehicle type. While these models are easy to implement, they often overlook the dynamic variability of driving conditions, such as acceleration, frequent stops, or changes in road gradient [11].

In contrast, microscopic models, such as the Comprehensive Modal Emissions Model (CMEM) developed by Barth et al. [12], provide a more precise and dynamic estimation of fuel consumption and pollutant emissions. This model incorporates parameters such as instantaneous speed, acceleration, engine load, and vehicle specific characteristics particularly for heavy-duty vehicles. It has been extensively validated in the work of Younglove and Scora [13], and has been successfully applied to assess the real-world impact of traffic congestion on emissions, as shown in studies of Barth and Boriboonsomsin [14].

In the CMEM model, speed is taken as a real parameter which is not at all realistic because of its variability related to road congestion, the typology of roads and temporal traffic conditions are changing in urban and peri-urban contexts as highlighted by Jabali et al. [15]. To address this limitation, we propose modeling speed as a fuzzy variable [16,17]**,** using linguistic terms such as *very slow*, *slow*, *moderate*, and *fast*, each associated with corresponding membership functions. This approach, grounded in fuzzy logic, enables a more realistic representation of traffic variability and uncertainty. Accordingly, in this paper, we introduce an enhanced version of the CMEM referred as Fuzzy-CMEM in which vehicle speed is treated as a fuzzy variable.

Based on this, we also develop a fuzzy multi-objective formulation of the GVRP, where fuel consumption is explicitly considered, and whose resolution is handled by the metaheuristic BicriterionAnt originally proposed by Iredi et al. [18]. This ant colony based method is capable of exploring optimal trade-offs between distance and energy consumption, even under uncertainty, as demonstrated in the work of Jabir et al. [19]. Liu et al. [20] assumed that carbon emissions are related to vehicle speed, load, and type, and developed an improved Ant Colony Optimization (ACO) algorithm to solve the problem. Kancharla and Ramadurai [21] included load, speed, and acceleration in fuel consumption estimation using driving cycles within vehicle routing problems, analyzing their impact on total fuel consumption and route selection. Early models that integrated speed into green VRP formulations were based on macroscopic approaches [9], in which consumption is expressed as a function of average speed along a road segment or route. These models typically use convex functions linking speed and fuel consumption. For instance, Kara et al. [5] introduced an energy-minimizing model where speed is optimized for each road segment, while Figliozzi [10] proposed a speed-based fuel consumption function tailored to different vehicle types. However, these approaches tend to overlook the dynamic variability of actual driving speeds and the effects of acceleration and deceleration cycles**.** To better reflect traffic realities, several authors have treated vehicle speed as a time-dependent variable. Jabali, et al. [15] analyzed the effects of congestion on travel speeds, demonstrating its direct impact on CO_2 emissions. Gupta et al. [22] studied a multi-objective capacitated green vehicle routing problem with fuzzy travel time–distance matrices and split deliveries represented as discrete packages. They employed fuzzy rule-based implication concepts for ranking and comparing fuzzy numbers with crisp values, leading to an expected value model. Based on this, a discrete hybrid fuzzy genetic algorithm was developed. Demir et al. [23] introduced a Pollution Routing Problem (PRP) that accounts for time-varying speeds, and adapted heuristics to optimize both departure times and routing based on allowable speeds. Kwon et al. [24] investigated heterogeneous fleet routing by incorporating speed profiles adjusted to each vehicle type. Liu, C.S. et al. [25] focus on a realistic variant of the Vehicle Routing Problem with Time Windows (VRPTW) in an urban context. The problem is time-dependent, meaning that vehicle speeds vary according to different times of the day (congestion periods). The main objective is to reduce delays and avoid congestion while respecting delivery time windows. Ye Chong et al. [26], in their study Optimization of Vehicle Paths Considering Carbon Emissions in a Time-Varying Road Network (TDGVRP), represent vehicle speed variation as a continuous function to make the model more consistent with real-world conditions and to promote the reduction of generated carbon emissions. They proposed a hybrid Genetic Algorithm–Simulated Annealing (GA-SA) for optimization. Fan, H. et al. [11] worked on the Time-Dependent Green Vehicle Routing Problem with Time Windows and Fuzzy Demand (TDGVRPTWFD)**.** Considering the time dependency of vehicle speed and the relationship between fuel consumption and vehicle type, speed, load, and road gradient, a stochastic programming model based on fuzzy credibility theory was formulated to minimize total cost and optimize vehicle routing under fuzzy demand. To solve the proposed problem, a chaotic genetic algorithm with a variable neighborhood search and rescheduling strategy was developed.

For greater precision, microscopic models such as the Comprehensive Modal Emissions Model (CMEM), developed in [12,13,14], this model is capable of simulating real-world driving cycles. The accuracy of CMEM makes it particularly relevant for the GVRP when vehicle speed is subject to spatio-temporal or stochastic variations. It has notably been employed in studies such as Turkensteen [27], which evaluates the precision of emission calculations relative to the granularity of speed data, and Kancharla and Ramadurai [19], who integrated it into a GVRP model sensitive to driving cycles. A more flexible approach consists of modeling speed as a fuzzy variable**,** allowing the capture of uncertainty linked to traffic conditions. This method is grounded in the principles of fuzzy logic introduced by Zadeh [16,17], enabling the use of linguistic terms like *slow, moderate, fast* to represent speed. El Fassi et al. [28] and Hussain & Allaoui [29] used fuzzy inference systems (FIS) to estimate fuel consumption based on uncertain speeds within an eco-driving framework. Nguyen et al. [30] incorporated fuzzy speed into an energy routing model for electric vehicles. Jabir et al. [19] explored the impact of fuzzy speeds within a GVRP solved using ant colony optimization.

Currently, very few studies explicitly and comprehensively incorporate fuzzy speed directly into fuel consumption models. Therefore, in this paper, we have chosen to use the Comprehensive Modal Emissions Model (CMEM)**,** a deterministic microscopic model developed to simulate fuel consumption and emissions based on detailed driving profiles such as instantaneous speed, acceleration, road grade, vehicle load, and more [12,13]. In this paper, we employ the Nearest Interval of Approximation (NIA) method as the defuzzification technique. This approach aligns with prior work by Demir et al. [23,31] and recent applications in multimodal routing or congestion-constrained contexts, such as those by Liu et al. [20] and Ye et al. [26]. Based on the above analysis, this paper focuses on the Fuzzy Multi-Objective Green Vehicle Routing Problem (FMOGVRP)**:** by considering vehicle speed as a fuzzy parameter and its relationship with fuel consumption, a fuzzy multi-objective optimization problem is formulated to minimize total distance and CO_2 emissions. The defuzzification process relies on the Nearest Interval of Approximation (NIA) of a fuzzy number, after which the sweep algorithm and the BicriterionAnt algorithm are applied respectively for customers clustering (potential routes) and constructing the problem's solutions.

The rest of paper is divided as follow: In section 2 we present some preliminary concepts which will be used in the sequel of the paper. In section 3, we present the problem statement and mathematical model. Section 4 is devoted in the methodology applied. In Section 5 is dedicated to the numerical Simulation. In Section 6 some discussions are presented. The paper is ended by concluding remarks in section 7.

II. Preliminaries

II.1. Fuzzy sets

Let X be a discourse universe non-empty. A fuzzy subset $\tilde{A} \in X$ is defined as follow: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$. Where $\mu_{\tilde{A}}: X \longrightarrow [0,1]$ is the membership function of the fuzzy set under consideration[16]. A level set α of $\tilde{A}$ is the set $\tilde{A}^{\alpha} = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$ [16]. In addition to a level subset, a fuzzy set can be characterized by its core, height and support. Let $H(\mathbb{R})$ the set of all normal, convex fuzzy numbers with compact support defined on $\mathbb{R}$, with membership function $\mu_{\tilde{A}}: X \longrightarrow [0,1]$. If $\tilde{a} \in H(\mathbb{R})$ then the level set α of $\tilde{a}$ is $[a_{\alpha}^{L}, a_{\alpha}^{U}]$ where a_{α}^{L} et a_{α}^{U} are continuous real valued functions on X. There are several types of $L - R$ fuzzy numbers when the reference functions are linear, we speak of triangular or trapezoidal fuzzy numbers.
Instead of working solely with a specific point such as a level subset, in this paper we focus on intervals, as they provide greater flexibility in decision-making. This is why, in this article, we will also rigorously use fuzzy concepts such as : The approximation interval operator given by $C: H(\mathbb{R}) \longrightarrow \mathcal{P}(\mathbb{R})$, the metric $d: H(\mathbb{R}) \longrightarrow [0, +\infty]$ and the Nearest Interval of Approximation (NIA) of $\tilde{a}$ in the sense of the metric d, for more details readers can consult [33, 34].

Algorithm 1: Nearest Interval of Approximation (NIA)
1. Read $\mu_{\tilde{a}}(x)$
2. Find $\inf\{x \in \mathbb{R} : \mu_{\tilde{a}}(x) \geq \alpha\} = a_{\alpha}^{L}$ and $\sup\{x \in \mathbb{R} : \mu_{\tilde{a}}(x) \geq \alpha\} = a_{\alpha}^{U}$, $\alpha \in (0,1]$
3. Calculate $\int_0^1 a^L(\alpha) d\alpha = N_{\alpha}^{L}$ and $\int_0^1 a^U(\alpha) d\alpha = N_{\alpha}^{U}$
4. Write $C_d(\tilde{a}) = [N_{\alpha}^{L}, N_{\alpha}^{U}]$

Ii.2. Combinatorial Optimization Problem

A multi-objective combinatorial optimization problem is a decision-making problem that consists in jointly optimizing a set of p often conflicting objective functions subject to a set of constraints with discrete (binary) variables, where : $F(x) = \left(f_1(x), f_2(x), \dots, f_p(x)\right)$ is a vector of objective functions [2]. The decision vector is $(x_1, x_2, \dots, x_n)$ and we note $D = \{x \in \{0,1\}^n : g_i(x) \leq 0, h_j(x) = 0; i = 1, \dots, m; j = 1, \dots, k\}$ the set of feasible solutions in the decision space. In the multi-objective framework, the decision-maker thinks rather in terms of evaluating a solution for each objective, and naturally places himself in the objective space. The image of a solution $x \in D$ in the objective space O is the point $(y_1, y_2, \dots, y_p)$ with $y_i = f(x_i), i = 1,2, \dots, p$; $Y = F(D)$ represents the solution in the goal space. The most widely accepted notion of optimality is the Pareto optimum [2]. From the above, this notion is extended to the fuzzy multi-objective combinatorial problem, assuming that:

(i) The set of feasible solutions D is nonempty, closed, and bounded, and therefore compact.
(ii) Let $\tilde{F}(x) = \left(\tilde{f}_1(x), \tilde{f}_2(x), \dots, \tilde{f}_p(x)\right)$ be a vector of fuzzy objective functions, where each $\tilde{f}_k \in H(\mathbb{R})$.
(iii) For any $x \in D$ and any $\alpha \in [0,1]$, level set α is given by $\left(\tilde{f}_k(x)\right)_{\alpha} = \left[f_{k,\alpha}^{L}(x), f_{k,\alpha}^{U}(x)\right]$, where $f_{k,\alpha}^{L}$ and $f_{k,\alpha}^{U}$ are continuous real-valued functions on D.
(iv) The fuzzy partial order relation $\preccurlyeq$ on $H(\mathbb{R})$ is defined by $\tilde{A} \preccurlyeq \tilde{B} \Leftrightarrow A_{\alpha}^{L} \leq B_{\alpha}^{L}$ and $A_{\alpha}^{U} \leq B_{\alpha}^{U}$ $\forall \alpha \in [0,1]$. The fuzzy Pareto dominance relation $\prec_P$ is the Pareto dominance induced by $\preccurlyeq$. For any $x, y \in D$, there exists at least one objective $i \in \{1, \dots, p\}$ such that Pareto dominance is defined as $x \prec_P y$ if and only if $\tilde{f}_i(x) \preccurlyeq \tilde{f}_k(y)$ and $\tilde{f}_k(x) \prec \tilde{f}_k(y)$.

II.3. Speed overview

In general, a vehicle's speed varies continuously for several reasons. Fan et al. [11] proposed that the relationship between vehicle speed (v) and time (t) can be expressed by the trigonometric function $v(t) = asin(\sigma t) + \delta$, where a, σ and δ are coefficients related to road conditions.

In urban area, vehicle speeds can be partitioned into periods T_1, T_2, T_3 and T_4 as illustrated in Figure 1.

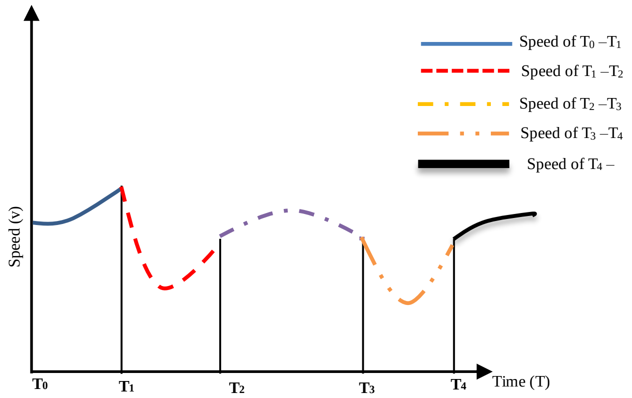

Figure 1. Daily variation trend of vehicle speed

III. Problem Statement And Mathematical Model

III.1. Comprehensive modal emission model (CMEM)

In arrange to degree the use of fuel and CO_2 emissions, an exact estimation strategy is connected. The CMEM is one of the estimation models of fuel utilization, which was created by Barth et al. 2005 [12]. The specified show comprises of three modules: motor control, motor speed and, fuel utilization rate.

The Outflow Rate (ER) $[g/s]$ for nursery gasses (such as CO, HC or NOx) is related to the use of fuel rate (FR) $[g/s]$, FR is a continuous, positive, and non-linear function, the calculation of FR is complex because it depends on a number of components. The calculation of FR is clarified in underneath connection (1)

$$FR = \left(z.N.W.\frac{\frac{P_t}{\varepsilon}+P_a}{\eta}\right) \times \gamma \quad (1)$$

Where z is the engine friction factor, N is the engine speed (radian per second (rps)), W $[liter]$ is the engine displacement, P_t $[watt]$ or $[kg\, m^2/s^3]$ is the total tractive power demand , ε $[-]$ is vehicle drive train efficiency, P_a $[watt]$ is the engine power demand associated with additional vehicle accessories such as air conditioner, η $[-]$ is a measure of efficiency for diesel engines and γ $[-]$ is a constant.

$$N = \frac{n_d n_g v}{R} \quad (2)$$

Where n_d is the differential ratio, n_g is the gear ratio and R is radius of the wheel. Moreover P_t $[kilowatt]$ is calculated as follow :

$$P_t = (Mav + Mgv\sin\theta + 0.5C_d\rho A v^3 + Mgv\cos\theta)10^{-3} \quad (3)$$

Where M $[kg]$ is the mass of the vehicle, v $[m/s]$ is speed, a $[m/s^2]$ is the acceleration, g $[\,9.81\, m/s^2]$ is the gravitational constant, θ $[radian]$ is the road angle, A $[m^2]$ is the surface area in front of the vehicle, ρ $[kg/m^3]$ is the air density, and C_r $[-]$ and C_d $[-]$ are the coefficients of rolling resistance and drag, respectively.

Thus,

$$FR = \gamma.z\frac{n_d n_g v}{R}.W + \frac{\gamma M v(a+g\sin\theta+gC_r\cos\theta+0.5C_d\rho A v^2)10^{-3}+\varepsilon P_a}{\eta.\varepsilon} \quad (4)$$

III.2. Problem Description

The Vehicle Routing Problem (VRP), proposed by Dantzig and Ramser [1], is an emblematic example of a combinatorial problem, recognized for its NP-hard complexity. Here, we present the formulation of FMOGVRP (Fuzzy Multi-Objective Green Vehicle Routing Problem) with homogeneous-capacity fleet vehicles which can be summarized as follows:

Let $G = (V, E)$ be an undirected graph, where $V = \{0\} \cup V_0$ represents a set of all nodes. Node 0 is the central repository from which all vehicles $k \in K = \{1,2, \dots, m\}$ depart; $V_0 = \{1,2, \dots, n\}$ denotes the set of customer nodes.

The set $E = \{(i,j): i \in V, j \in V\}$ is the edge set, d_{ij} represents the distance between nodes i and j. All customers have a specific demand $l_i \geq 0$ to be served by a vehicle k of capacity Q. The speed of each vehicle is expressed by trapezoidal fuzzy variable $v = (v^1, v^2, v^3, v^4)$.

The binary decision variables x_{ij}^k are equal to 1 if vehicle k visits node j directly after node i, and 0 otherwise. The main objectives of the problem is to minimize the total travelled distance of all routes driven by vehicles k and to minimize the fuel consumption and CO_2 emissions. Based on the above, we formulate the following assumptions:

- **H1**: fuel consumption occur with a warmed-up engine: according [9] total fuel consumption are given by $FR_{total} = FR_{cold\ start} + FR_{Hot}$ and if $FR_{cold\ start} = 0 \Rightarrow FR_{total} = FR_{Hot}$
- **H2**: The vehicles used belong to the Heavy-Duty Vehicle (HDV): 3.5-7.5 tonnes
- **H3**: The road gradient is zero degree.
- **H4**: Vehicle speed is modeled as a trapezoidal fuzzy number $v = (v^1, v^2, v^3, v^4)$.

IV. Methodology

This fuzzy approach enables the modeling of imprecision associated with real traffic conditions. Thus, speed is no longer treated as a fixed parameter but as flexible variable whose value can fluctuate across several categories. This uncertainty is most often epistemic in nature, arising from a lack of information or imprecise data rather than from purely random phenomena. Deterministic methods, based on the assumption of constant speeds, are unable to reflect this variability and may therefore lead to solutions with limited robustness. While stochastic models allow uncertainty to be explicitly taken into account, they nevertheless require the availability of reliable probability distributions as well as large volumes of historical data conditions that are rarely met in the context of urban freight transportation. By contrast, the fuzzy approach provides a flexible modeling framework capable of representing speed fluctuations through fuzzy numbers, without relying on restrictive probabilistic assumptions. To this end, we consider a trapezoidal fuzzy number $\tilde{v} = (v^1, v^2, v^3, v^4)$ describing vehicle travel speed with the linguistic terms {very slow (VS), slow(S), moderate (M), fast (F), and very fast (VF)}, whose membership functions are defined as follows:

$$\mu_{\tilde{v}}(x) = \begin{cases} 0 & if \quad x \le v^1 \\ \frac{x-v^1}{v^2-v^1} & if \quad v^1 < x \le v^2 \\ 1 & if \quad v^2 \le x \le v^3 \\ \frac{v^4-x}{v^4-v^3} & if \quad v^3 < x \le v^4 \\ 0 & if \quad x > v^4 \end{cases} \quad (5)$$

where v^1, v^2, v^3, v^4 are real numbers and $v^1 < v^2 < v^3 < v^4$. In this formulation, the interval $[v^2, v^3]$ corresponds to the most plausible operational speeds under typical traffic conditions, whereas $[v^1, v^4]$ defines the range of extreme, yet still feasible speed values. The performance of the fuzzy modeling framework largely depends on the specification of the membership functions associated with fuzzy speeds, which are based on empirical observations and expert knowledge as reported in the literature [Ref]. Inadequate calibration of these functions may introduce subjective bias and undermine the reliability of the resulting outcomes.

IV.1. A Novel adaptation of CMEM and mathematical formulation of FMOGVRP

According to the existing categorization of factors that influence fuel consumption of vehicles directly, speed is part of the category of factors that are difficult to control and unpredictable but measurable. Whereas existing studies generally rely on deterministic or stochastic models to evaluate fuel consumption and/or CO_2 emissions [38, 39], in this paper we introduce the new concept Fuzzy-CMEM in order to calculate the fuel consumption of a vehicle k that moves from point i to point j using the speed as a fuzzy parameter while preserving the analytical structure of the CMEM.

The Fuzzy-CMEM model assumes that all vehicle related parameters, except for speed such as the vehicle mass M or engine characteristics P_t, P_a etc. are perfectly known and remain constant throughout the process, even though this scenario may seem restrictive, the Fuzzy-CMEM model allows us to enrich the classical modeling by making it more realistic in the face of real-world road traffic uncertainties (variable traffic, non-constant speed limits, road conditions).

The Fuzzy-CMEM model is presented below: Let us incorporate fuzzy speed $\tilde{v}$ into the relation (4) ,

$$\widetilde{FR}(v_{ij}) = \gamma . zW \frac{n_d n_g \tilde{v}_{ij}}{R} d_{ij} + \frac{\gamma M \tilde{v}_{ij}\left(a + g\sin\theta + gC_r\cos\theta + 0.5C_d\rho A\tilde{v}_{ij}^2\right)10^{-3}d_{ij} + \varepsilon P_a d_{ij}}{\eta . \varepsilon} \quad (6)$$

As a trapezoidal fuzzy number $\widetilde{FR}$, can be written as : $\widetilde{FR} = \left(s(v^1), s(v^2), s(v^3), s(v^4)\right)$ with the membership function defined by :

$$\mu_{\widetilde{FR}}(x) = \begin{cases} 0 & if \quad x \le s(v^1) \\ \frac{x-s(v^1)}{s(v^2)-s(v^1)} & if \quad s(v^1) < x \le s(v^2) \\ 1 & if \quad s(v^2) \le x \le s(v^3) \\ \frac{s(v^4)-x}{s(v^4)-s(v^3)} & if \quad s(v^3) < x \le s(v^4) \\ 0 & if \quad x > s(v^4) \end{cases} \quad (7)$$

Setting that $\tau_{ij} = a + g\sin\theta_{ij} + gC_r\cos\theta_{ij}$ and $\beta_{ij} = 0.5C_d\rho A$, we can then rewrite relation (6) as follows:

$$\widetilde{FR}(v_{ij}) = \gamma . zW \frac{n_d n_g \tilde{v}_{ij}}{R} d_{ij} + \frac{\gamma M \tilde{v}_{ij}\left(\tau_{ij} + \beta_{ij}\tilde{v}_{ij}^2\right)10^{-3}d_{ij} + \varepsilon P_a d_{ij}}{\eta . \varepsilon} \quad (8)$$

Based on the above, we can formulate our FMOGVRP optimization problem (P1) as follows:

$$f_1 = \min \sum_{k=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}^k \quad (9)$$

$$\tilde{f}_2 = \min \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} \left(\gamma . zW \frac{n_d n_g \tilde{v}_{ij}}{R} d_{ij} + \frac{\gamma M \tilde{v}_{ij}\left(\tau_{ij} + \beta_{ij}\tilde{v}_{ij}^2\right)10^{-3}d_{ij} + \varepsilon P_a d_{ij}}{\eta . \varepsilon} \right) x_{ij} \quad (10)$$

Subject to:

$$\sum_{k=1}^{v} \sum_{j=0}^{n} x_{ij}^k = 1 \quad \text{if } i \in V\backslash\{0\} \quad (11)$$

$$\sum_{i=0}^{n} x_{ij}^k = \sum_{j=0}^{n} x_{ji}^k \quad \text{if } l \in V\backslash\{0\}, k = 1, \dots, m \quad (12)$$

$$\sum_{j=1}^{n} x_{0j}^k = 1 \quad \text{if } k = 1, \dots, m \quad (13)$$

$$\sum_{i=1}^{n} x_{i0}^k = 1 \quad \text{if } k = 1, \dots, m \quad (14)$$

$$\sum_{i=0}^{n} \sum_{j=0}^{n} s_i x_{ij}^k + \sum_{i=0}^{n} \sum_{j=0}^{n} t_{ij} x_{ij}^k \le T \quad k = 1, \dots, m \quad (15)$$

$$\sum_{i\in U}^{v} \sum_{j\in U}^{n} x_{ij}^k \ge \sum_{j=1}^{n} x_{ij}^k \quad , \ \forall U \subset V\backslash\{0\}, l \in U \ ; \ k = 1, \dots, m \quad (16)$$

$$\sum_{i=1}^{n} q_i \left(\sum_{j=0}^{n} x_{ij}^k \right) \le Q \quad k = 1, \dots, m \quad (17)$$

$$x_{ij}^k \in \{0,1\} \quad (18)$$

The set of feasible solutions is given by $D = \{x_{ij}^k \in \{0,1\} : (11) - (17) \text{ are satisfied}, i = 1, \dots, n \ ; j = 1, \dots, n \ , k = 1, \dots m\}$.

Function (9) defines the objective function aimed at minimizing the total distance, while function (10) defines the objective function targeting the minimization of total CO_2 emissions. The constraints can be interpreted as follows: Expression (11) ensures that each customer is visited exactly once, (12) stipulates that if a vehicle k arrives at customer j it must depart from j once finish to server, (13) and (14) require each vehicle k to return to the depot at the end of its route, (15) enforces the maximum allowed route duration T, (16) eliminates sub-tours to guarantee route connectivity, (17) there is also the vehicle capacity constraint and equation (18) specifies that the decision variables are binary.

Theorem 1 : Under assumptions (i)-(iv), the fuzzy multi-objective problem $\min_{x \in D} \tilde{F}(x)$ defined by (P1) is mathematically well posed and admits at least one fuzzy Pareto-optimal solution.

Proof : By (ii)-(iii), for any $x \in D$, $\tilde{F}(x) \in H(\mathbb{R})$ thus the problem defines a well-defined fuzzy application $\tilde{F}: D \longrightarrow H(\mathbb{R})$. According to assumption (iii), for $\alpha \in [0,1]$ the real functions $F_\alpha(x) = \left(\left[f_{1,\alpha}^L(x), f_{1,\alpha}^U(x)\right], \dots, \left[f_{p,\alpha}^L(x), f_{p,\alpha}^U(x)\right]\right)$ are continuous over D. Thus, for all α, the fuzzy problem induces a continuous deterministic multi-objective problem. Hence, for each $\alpha \in [0,1]$, there exists at least one Pareto-optimal solution $x_\alpha^* \in D$ for the induced deterministic problem. In other words, assume, for the sake of contradiction, that no fuzzy Pareto-optimal solution exists. Then, for all $x \in D$, there exists $y \in D$ such that $\tilde{F}(x) \prec_P \tilde{F}(y)$. However, by the definition of $\prec_P$ and (iv), this implies that $F_\alpha(y) \prec_P F_\alpha(x) \quad \forall\, \alpha \in [0,1]$ which contradicts the existence, for each α of at least one deterministic Pareto-optimal solution. Hence, there exists $x^* \in D$ such that $\nexists\, y \in D$ satisfies $F(y) \prec_P F(x)$.

IV.2. Defuzzification of FMOGVRP

Contrary to commonly employed methods defuzzification to a single point in the literature such as the Centre of Sums (COS), Centre of Gravity (COG), Weighted Average, and Maxima approaches [36], the Nearest Interval of Approximation (NIA) is employed here as a defuzzification strategy because it allows partial preservation of uncertainty information by maintaining the lower and upper bounds at each α level. This approach enables us to transform the FMOGVRP into an equivalent deterministic MOGVRP while retaining the range of plausible values induced by speed uncertainty.

Let us apply Algorithm 1 to the membership function $\mu_{\tilde{v}}(x)$ given by expression (5) and we consider $\alpha \in [0,1]$,

$$\frac{x - v^1}{v^2 - v^1} = \alpha \Longrightarrow x - v^1 = \alpha(v^2 - v^1)$$

$$\Longrightarrow x = \alpha(v^2 - v^1) + v^1 \qquad (19)$$

$$\frac{v^4 - x}{v^4 - v^3} = \alpha \Longrightarrow v^4 - x = \alpha(v^4 - v^3)$$

$$\Longrightarrow x = v^4 - \alpha(v^4 - v^3) \qquad (20)$$

Hence, from (18) and (19), we respectively have

$$N_\alpha^L = \int_0^1 (\alpha(v^2 - v^1) + v^1) d\alpha = \int_0^1 \alpha(v^2 - v^1)\, d\alpha + \int_0^1 v^1\, d\alpha$$

$$= (v^2 - v^1)\int_0^1 \alpha\, d\alpha + v^1 \int_0^1 d\alpha = \frac{1}{2}(v^2 - v^1) + v^1$$

And

$$N_\alpha^U = \int_0^1 \left(v^4 - \alpha(v^4 - v^3)\right) d\alpha = \int_0^1 v^4\, d\alpha - \int_0^1 \alpha(v^4 - v^3)\, d\alpha$$

$$= v^4 \int_0^1 d\alpha - (v^4 - v^3)\int_0^1 \alpha\, d\alpha$$

$$= v^4 - \frac{1}{2}(v^4 - v^3)$$

Thus, the nearest interval of approximation is:

$$C_d(\tilde{v}) = \left[\frac{1}{2}(v^2 - v^1) + v^1, v^4 - \frac{1}{2}(v^4 - v^3)\right] \qquad (21)$$

Instead of taking the lower bound or upper bound, in this paper we consider the mean of the nearest interval of approximation as a deterministic result because the mean of the interval provides a real value that takes into account both the extremes and the core. This allows avoiding a purely optimistic N_α^L or even pessimistic N_α^U option.
Thus mean, the expression (21) is given by:

$$Av\left(C_d(\tilde{v})\right) = \frac{\frac{1}{2}(v^2 - v^1) + v^1 + v^4 - \frac{1}{2}(v^4 - v^3)}{2} \qquad (22)$$

However, NIA entails a certain loss of information, as it reduces the full structure of fuzzy parameters to real intervals. This effect can influence the interpretation of Pareto dominance within a strictly fuzzy framework, particularly in multi-objective decision-making contexts. Nevertheless, employing this approximation represents a deliberate trade-off between uncertainty representation and computational tractability.

By substituting (22) into (10), we obtain a deterministic counterpart of the fuzzy objective function f_2 that describes the fuel consumption of Problem (MOGVRP) expressed by:

$$\min \sum_{k\in K}\sum_{i\in V}\sum_{j\in V}\left(\gamma.zW\frac{n_d n_g Av(C_d(\tilde{v}))}{R}d_{ij} + \frac{\gamma MAv(C_d(\tilde{v}))\left(\tau_{ij}+\beta_{ij}\left(Av(C_d(\tilde{v}))\right)^2\right)10^{-3}d_{ij}+\varepsilon P_a d_{ij}}{\eta.\varepsilon}\right)x_{ij} \quad (23)$$

The deterministic MOGVRP can be written as:

$$f_1 = \min \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij}^{k}$$

$$f_2 = \min \sum_{k\in K}\sum_{i\in V}\sum_{j\in V}\left(\gamma.zW\frac{n_d n_g Av(C_d(\tilde{v}))}{R}d_{ij} + \frac{\gamma MAv(C_d(\tilde{v}))\left(\tau_{ij}+\beta_{ij}\left(Av(C_d(\tilde{v}))\right)^2\right)10^{-3}d_{ij}+\varepsilon P_a d_{ij}}{\eta.\varepsilon}\right)x_{ij}$$

Subject to:

$$\sum_{k=1}^{v}\sum_{j=0}^{n} x_{ij}^{k} = 1 \quad \text{if } i \in V\backslash\{0\}$$

$$\sum_{i=0}^{n} x_{ij}^{k} = \sum_{j=0}^{n} x_{ji}^{k} \quad \text{if } l \in V\backslash\{0\}, k = 1,\dots,m$$

$$\sum_{j=1}^{n} x_{0j}^{k} = 1 \quad \text{if } k = 1,\dots,m$$

$$\sum_{i=1}^{n} x_{i0}^{k} = 1 \quad \text{if } k = 1,\dots,m$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} s_i x_{ij}^{k} + \sum_{i=0}^{n}\sum_{j=0}^{n} t_{ij}x_{ij}^{k} \le T \quad k = 1,\dots,m$$

$$\sum_{i\in U}^{v}\sum_{j\in U}^{n} x_{ij}^{k} \ge \sum_{j=1}^{n} x_{ij}^{k} \quad , \forall U \subset V\backslash\{0\}, l \in U\ ;\ k = 1,\dots,m$$

$$\sum_{i=1}^{n} q_i\left(\sum_{j=0}^{n} x_{ij}^{k}\right) \le Q \quad k = 1,\dots,m$$

$$x_{ij}^{k} \in \{0,1\}$$

Theorem 2: (Characterization of efficient solutions to the deterministic MOGVRP using the reference frame)
Let O be the objective space of problem (P1), $y^* \in O$ is the efficient solution. The following statements are equivalent:

a) $y^* \in O$ is an efficient (Pareto optimal) solution of (P1)
b) There is no $y^* \in O$ such that $y^* \in O$ is located in the dominated solution area of the reference frame of y.
c) There is no $y \in O$ such that z is located in the dominant solution area of the reference frame of y^*.

Proof: (a)⇒(b) : Suppose that y^* is efficient for (P1), hence, by definition of efficiency, there not exist $y \in O$ such that $y \le y^*$, but y^* is in the dominated zone of y, which is exactly equivalent to $y \le y^*$. Therefore, there not exist $y \in O$, such that y^* is in the dominated zone of z. Hence (b).

(b)⇒(a): Suppose (b); there is no $y \in O$ with y^* in the dominated set of y. Since y^* is in the dominated set of y, i.e. $y \le y^*$, there is no $y \in O$, such that $y \le y^*$. By definition, this means that y^* is efficient. Therefore, (a) is true.

Formulations (b) and (c) express in words two ways of saying the same thing: There is no $y \in O$ that dominates y^*. Indeed,

(i) y^* is in the dominated solution area of y, which means that $y \le y^*$i.e. y dominates y^*
(ii) y is in the dominant solution area of y^* also means $y \le y^*$.

Thus by (i)-(ii) we said that (b) and (c) are equivalent to each other. Therefore, by transitivity, we say that (a) implies (c) ■

For the equivalent deterministic model MOGVRP the necessary conditions of optimality given by Karush-Kuhn-Tucker (KKT), unfortunately do not apply directly to a combinatorial problem, because the latter contains binary variables (NP-Hard), hence the need to use heuristics and metaheuristics to find solutions to the problem. In the following lines, in this paper relies on the consecutive application of the sweep heuristic for partitioning customers into different clusters (potential routes) based on their polar angle relative to the central depot [37], our choice falls on the sweep heuristic because, in addition to its simplicity, it has a lower computational complexity $O(n \log n)$ compared to Clarke-Wright $O(n^2 \log n)$ and insertion heuristics $O(n^2)$ [38] making it better suited for initializing solutions in a GVRP.

Followed by the BicriterionAnt metaheuristic for optimizing the route costs according to the two objective functions f_1 and f_2 , directly applying the concept of Pareto dominance in the selection of generated solutions [18]. Even though the MOGVRP is solved deterministically, the resulting Pareto solutions are directly derived from the initial fuzzy formulation. The resulting Pareto front can therefore be interpreted as a robust approximation of the fuzzy Pareto front, where dominance relationships are assessed with respect to the actual value intervals rather than fixed predetermined values.

V. NUMERICAL SIMULATION

V.1. Presentation of data used

In this paper, five types of vehicle speed are considered, as shown in Table 1 below, with information adapted from [39].

TABLE I
PRESENTATION AND DESCRIPTION OF THE SPEED CATEGORIES USED

Types of speed	**Description**	**Trapezoidal fuzzy number**
Very slow (VS)	0 to 30 Km/h	(0, 0, 10,30)
Slow (S)	20 to 60 Km/h	(20, 30, 50,60)
Moderate (M)	50 to 90 km/h	(50,60,80, 90)
Fast (F)	80 to 120 km/h	(80,90,110,120)
Very (VF)	110 to 160+ km/h	(110,120,160,180)

Table 2 below presents the different accelerations associated with each speed type. Based on this, we categorized the acceleration corresponding to each speed type. This represents a typical classification used in CMEM-based modeling within the context of GVRP [39].

TABLE II
CATEGORIZATION OF ACCELERATIONS ASSOCIATED WITH SPEEDS

Types of speed	Characteristic acceleration $[m/s^2]$
Very slow (VS)	0 to 0.5
Slow (S)	0,5 to 1,0
Moderate (M)	1,0 to 1,8
Fast (F)	1,8 to 3,0
Very fast (VF)	$> 3,0$

In order to represent the variability of fuel consumption in relation to speed as reported in [39], we adapted on experimental basis [40] the fuel consumption as a trapezoidal fuzzy number for research purposes, as can be seen in Table 3.

TABLE III
FUEL CONSUMPTION LEVEL

Types of Consumption	Trapezoidal fuzzy number
Low	(3.0, 3.5, 4.5,5.0)
Medium	(4.0, 4.5, 6.5,7.0)
High	(6.0, 7.0, 9.0,10.0)
Medium-high	(7.0, 8.0,10.0, 11.0)
Very high	(9.0,11.0,14.0 16.0)

The different parameters used for the CMEM model are provided in Table 4 below.
We use the heavy-duty vehicle type with a gross vehicle weight of 3.5-7.5 tonnes, which is commonly employed in energy consumption models for vehicles [21].

TABLE IV
DESCRIPTION OF PARAMETERS USED IN CMEM

Parameters	Description	Value used	Source
v	Speed in $[m/s]$	-	Author
a	Acceleration in $[m/s^2]$	-	Author
M	Gross Vehicle weight in[kg]	6350	[21] [39]
g	Gravitational constant $[m/s^2]$	9.81	
θ	Road grade angle [degrees]	0	
ρ	Air density in $[kg/m^3]$	1.2041	
A	Frontal surface area in $[m^2]$	3.341	
C_d	Coefficient of aerodynamic drag [-]	0. 9	
C_r	Coefficient of rolling resistance [-]	0.07	
ε	Vehicle drive train efficiency [-]	0.4	
P_a	Engine power demand associatted with running losses of the engine [-]	0	
γ	Fuel to air mass ratio [-]	0.0667	
z	Engine friction factor [-]	0.2	
R	Rayon [m]	0.35	
n_d	Differential	4.2	
n_g	1ere vitesse $[m/s]$	3.6	
W	Engine displacement [litre]	5.83	
η	Efficiency for diesel engines [-]	0.45	

We select and modify the instance P-n55-k10 Q=115 from Augerat 1995 VRP Dataset [41], in order to evaluate the effect of vehicle speed on the optimal routes in different scenarios. The major modifications are: i) a new vehicle type having a capacity multiplied by thirty that of the original is introduced. ii) the demand of customers is multiplied by twenty. The both modifications are done because of vehicle type used is HVD (3.5-7.5 tonnes), see Table 5.

TABLE V
CUSTOMERS DATA

Node	Coord X	Coord Y	Demand	Node	Coord X	Coord Y	Demand	Node	Coord X	Coord Y	Demand
0	40	40	0	**19**	62	48	300	**38**	47	66	480
1	22	22	360	**20**	66	14	440	**39**	30	60	320
2	36	26	520	**21**	44	13	560	**40**	30	50	660
3	21	45	220	**22**	26	13	240	**41**	12	17	300
4	45	35	600	**23**	11	28	120	**42**	15	14	220
5	55	20	420	**24**	7	43	540	**43**	16	19	360
6	33	34	380	**25**	17	64	280	**44**	21	48	340
7	50	50	300	**26**	41	46	360	**45**	50	30	420
8	55	45	320	**27**	55	34	340	**46**	51	42	540
9	26	59	580	**28**	35	16	580	**47**	50	15	380
10	40	66	520	**29**	52	26	260	**48**	48	21	400
11	55	65	740	**30**	43	26	440	**49**	12	38	100
12	35	51	320	**31**	31	76	500	**50**	15	56	440
13	62	35	240	**32**	22	53	560	**51**	29	39	240
14	62	57	620	**33**	26	29	540	**52**	54	38	380
15	21	24	160	**34**	50	40	380	**53**	55	57	440
16	21	36	380	**35**	55	50	200	**54**	67	41	320
17	33	44	400	**36**	54	10	240				
18	9	56	260	**37**	60	15	280				

The algorithms of Sweep and BicriterionAnt has been implemented in Python language on a HP Laptop with CPU Intel® Core™i5-8365U, 1.90 GHz with 8 GB of RAM. Details of parameters used for BicriterionAnt are reported in Table 6.

TABLE VI
PARAMETERS DESCRIPTION FOR THE BICRITERIONANT ALGORITHM

Parameters	Value	Description	Source
m	7	The number of ants	[2][42]
π	1	Weight for pheromone level	
ω	2	Weight for heuristics information	
ρ	0.2	Pheromone evaporation constant	
$Iteration$	100	Number of iterations	

V.2. Results

The graph in Figure 2 illustrates the spatial distribution of 54 customers spread across a two-dimensional geographic area. Each customer $i \in \{1,2,\dots,54\}$ is represented by a node identified by its Cartesian coordinates (x_i, y_i).

A central depot (x_0, y_0) is also depicted. This type of graph serves as a visual basis for applying clustering algorithms for vehicle routing.

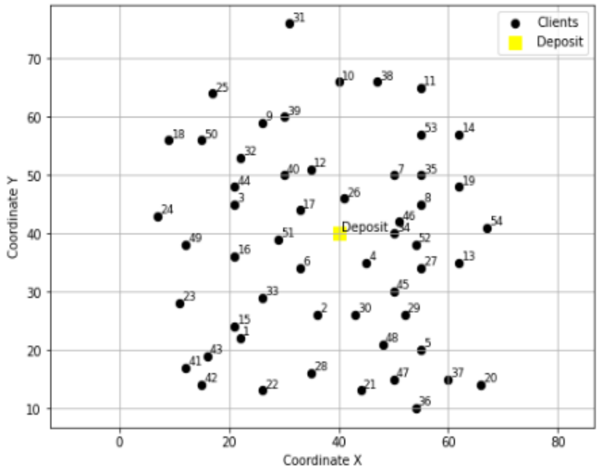

Figure 2. Spatial distribution of customers

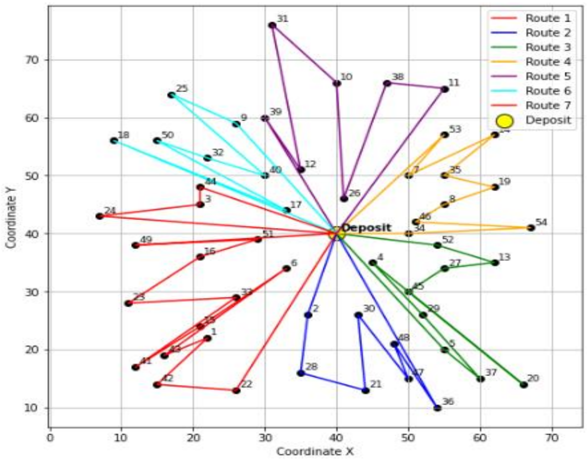

Figure 3. Result of the sweep algorithm

Figure 3 above shows the resolution of the system of constraints (11) - (18) of Problem (P1), which enabled us to find the initial solutions to the problem by clustering customers (potential routes). As (P1) is an NP-hard problem, we used the Sweep heuristic to achieve so we can show how customers belonging to the same group (or route) are represented by distinct color, allowing visualization of the different service zones assigned to vehicles.

This representation highlights the geographical logic of the sweep algorithm: customers close in angular position are clustered together, which helps to minimize the travel distance while respecting capacity constraints.

V.2.1. Deffuzification and Solution of Problem FMOGVRP

A. Deffuzification phase

As mentioned above, in this phase we will use the Nearest Interval of Approximation (NIA). First, we will verify its effectiveness by a comparative analysis of the NIA and COG methods was conducted using the same dataset represented by trapezoidal fuzzy numbers describing all the speed types considered as we can see in the table 7 below.

TABLE VII
COMPARIISON BETWEEN NIA AND COG

Trapezoidal fuzzy number	Types of speed	NIA			COG	Gap (NIA-COG)
		N_α^L	N_α^U	$Av(C_d(\widetilde{v}))$		
(0, 0, 2, 5)	Very slow (0-30 km/h)	0	3,5	1,75	1,5	0,3
(1, 3, 4, 6)	Very slow (0-30 km/h)	2	5	3,5	3,5	0,0
(2, 4, 5, 7)	Very slow (0-30 km/h)	3	6	4,5	4,5	0,0
(3, 5, 6, 8)	Very slow (0-30 km/h)	4	7	5,5	5,5	0,0
(4, 6, 7, 9)	Very slow (0-30 km/h)	5	8	6,5	6,5	0,0
(5, 7, 8, 10)	Very slow (0-30 km/h)	6	9	7,5	7,5	0,0
(8, 10, 15, 18)	Slow (20-60 Km/h)	9	16,5	12,75	12,7	0,1
(10, 15, 18, 22)	Slow (20-60 Km/h)	12,5	20	16,25	16,3	0,1
(12, 17, 20, 25)	Slow (20-60 Km/h)	14,5	22,5	18,5	18,5	0,0
(14, 18, 22, 28)	Slow (20-60 Km/h)	16	25	20,5	20,3	0,2
(16, 20, 25, 30)	Slow (20-60 Km/h)	18	27,5	22,75	22,7	0,1
(18, 22, 28, 35)	Slow (20-60 Km/h)	20	31,5	25,75	25,5	0,3
(28, 32, 38, 45)	Moderate (50-90 km/h)	30	41,5	35,75	35,5	0,3
(30, 35, 42, 48)	Moderate (50-90 km/h)	32,5	45	38,75	38,7	0,1
(35, 40, 45, 50)	Moderate (50-90 km/h)	37,5	47,5	42,5	42,5	0,0
(38, 43, 48, 55)	Moderate (50-90 km/h)	40,5	51,5	46	45,8	0,2
(40, 46, 52, 60)	Moderate (50-90 km/h)	43	56	49,5	49,3	0,2
(45, 50, 58, 65)	Moderate (50-90 km/h)	47,5	61,5	54,5	54,3	0,2
(55, 60, 70, 80)	Fast (80-120 km/h)	57,5	75	66,25	65,8	0,4
(60, 70, 80, 90)	Fast (80-120 km/h)	65	85	75	75,0	0,0
(65, 75, 85, 95)	Fast (80-120 km/h)	70	90	80	80,0	0,0
(70, 80, 90, 100)	Fast (80-120 km/h)	75	95	85	85,0	0,0
(75, 85, 95, 105)	Fast (80-120 km/h)	80	100	90	90,0	0,0
(80, 90, 100, 110)	Fast (80-120 km/h)	85	105	95	95,0	0,0
(95, 100, 110, 120)	Very fast (110-160+ km/h)	97,5	115	106,25	105,8	0,4
(100, 110, 120, 130)	Very fast (110-160+ km/h)	105	125	115	115,0	0,0
(105, 115, 125, 135)	Very fast (110-160+ km/h)	110	130	120	120,0	0,0
(110, 120, 130, 140)	Very fast (110-160+ km/h)	115	135	125	125,0	0,0

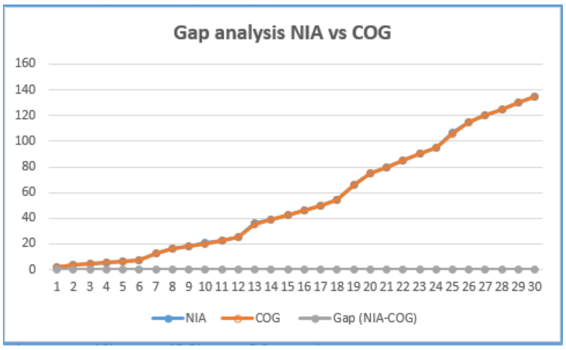

Figure 4. Comparison curve NIA vs COG

In Figure 4, the representative curves of both methods are superimposed to visualize any potential differences. Following this superposition, it became clear that the NIA and COG curves perfectly overlap, with no visible divergence across the entire studied interval. From a numerical standpoint, this coincidence of results is confirmed by calculating the gap (absolute difference) between the values produced by each method at every point. For each corresponding data pair (NIA_i, COG_i) the $Gap|NIA_i - COG_i|$ was evaluated. In all cases, the gap is insignificant close to zero, which means that $\forall\, i, NIA_i \equiv COG_i$, this indicates that the two methods are equivalent in the defuzzification of data.

B. Solution of Problem (P1)

In the context of modeling vehicle behavior on a road network, we associate a characteristic speed type to each road segment. Due to the lack of real data, we use a controlled random assignment of speed types. Thus, each road segment is randomly assigned a speed belonging to one of the five linguistic classes considered as we can see in the table 1. This random distribution is performed according to a uniform probability law because all possible outcomes of a random experiment have the same probability of being chosen. The use of this method allows us to introduce variability into speed profiles, simulate realistic scenarios where speed is not uniform across the entire network. After route creation and defuzzification phases, we apply now the BicriterionAnt metaheuristic on the equivalent deterministic MOGVRP according to the parameters reported in the table 6.

TABLE VIII
OPTIMAL ROUTES OBTAINED BY BICRITERIONANT

		Under fuzzy aspect	
Vehicles	Routes	f_1	f_2
1	0-6-33-15-1-22-42-41-43-23-49-16-51-0	107.7	46.4
2	0-30-48-47-36-21-28-2-0	78.6	11.4
3	0-52-27-13-20-37-5-29-45-4-0	85.1	35.1
4	0-34-46-8-54-19-14-53-35-7-0	80.6	38.9
5	0-12-39-31-10-38-11-26-0	96.6	18.6
6	0-17-32-50-18-25-9-40-0	81.4	41.6
7	0-24-3-44-0	66.9	4.7

The Table 8, show us the results from the BicriterionAnt algorithm, applied to the deterministic MOGVRP, provide a set of Pareto optimal routes that aim to reconcile two main objectives: reducing the total traveled distance and minimizing energy consumption.

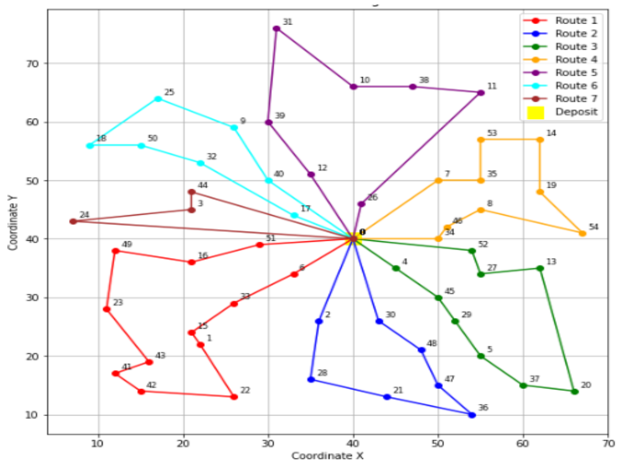

Figure 5. Routes optimized by BicriterionAnt

Figure 5 shows us the graphical layout of the route optimisation from Table 8. However, Table 9 below presents, for each road segment belonging to the optimal routes, the randomly assigned speed. Each row corresponds to a specific segment, while the columns represent the speed classes {very slow (VS), slow(S), moderate (M), fast (F), very fast (VF)}.

The obtained Pareto front exhibits a relatively high spacing value, indicating an irregular distribution of non-dominated solutions. While the trade-off between cost and emissions is clearly captured, the uneven spacing suggests that certain compromise regions are under-represented, leaving room for improving solution diversity. From the decision-maker's perspective, firstly, this Pareto front can be used as a means to identify compromise solutions that balance environmental impact and operational cost under conditions of uncertain speed. Secondly, the extreme solutions on the Pareto front can be interpreted as policy-focused scenarios, such as emission-minimization strategies under strict environmental regulations or cost-oriented strategies when economic constraints dominate.

TABLE IX
SPEED DISTRIBUTION ON OPTIMAL ROUTES

Routes \ Speed	VS	S	M	F	VF
Route 1	16-51-0	0-6-33-15		15-1-22-42-41-43	43-23-49-16
Route 2		47-36-21	0-30-48-47	21-28-2-0	
Route 3			13-20-37-5	0-52-27-13	5-29-45-4-0
Route 4	0-34-46-8		8-54-19	19-14-53	53-35-7-0
Route 5	38-11-26-0	39-31	0-12	12-39 & 31-10-38	
Route 6			25-9-40-0	0-17-32	32-50-18-25
Route 7	3-0	24-3	0-24		

In the following lines, we will attempt to carry out a comparative analysis between the fuzzy and deterministic CMEM models using the objective function f_2 .

TABLE X
COMPARATIVE ANALYSIS OF FUZZY AND FIXED VEHICLE SPEEDS ON f_2

	f_2						
Routes	**Fuzzy speed**	**Speed (30 km/h)**	**Speed (60 km/h)**	**Speed (120 km/h)**	**Fuzzy speed vs speed 30km/h**	**Fuzzy speed vs speed 60km/h**	**Fuzzy speed vs speed 120km/h**
1	46.4	14.8	11.4	86.6	31.9 %	24.6%	186.6%
2	11.4	7.1	5.3	42.0	62.3 %	46.5%	368.4%
3	35.1	9.5	7.0	55.6	27.0 %	19.9%	158.3%
4	38.9	8.9	6.7	52.6	22.9 %	17.2%	135.2%
5	18.6	10.7	8.0	63.1	57.5 %	43.0%	339.8%
6	41.6	7.7	7.0	70,1	18.5 %	16.8%	168.5%
7	4.7	7,8	7.4	70,8	165.9%	157.4%	1506.4%

VI. DISCUSSION

From the following formula $\% = \frac{fixed\ speed}{fuzzy\ speed} \times 100$, we were able to compare the results of objective function f_2 when the speed is fuzzy and when the speed is fixed, Table 10 shows that when the vehicle speed is fixed at 30 km/h, fuel consumption are significantly lower compared to the fuzzy speed model across all routes on average, the speed of 30 km/h represents between 18% and 62% of the fuzzy speed, except for route 7 where it greatly exceeds it (due to the very low fuzzy speed). Similarly, when the speed is set at 60 km/h, the fuel consumption lower on average it generally represents 17% to 46% of the fuzzy speed. This demonstrates that using a fixed speed in the CMEM model severely underestimates fuel consumption compared to our Fuzzy-CMEM model. Conversely, when the speed is set to 120 km/h, fuel consumption exceeds that of the fuzzy speed model, by more than +100%, therefore, a speed of 120 km/h is completely incompatible with an urban distribution context; it exceeds the fuzzy speed in an extreme way, this confirming that categorizing speed as "high" leads to very high fuel usage.
We consider these results as an illustrative validation of the approach proposed in this paper rather than a generalization, since problems of different sizes were not addressed.

VII. CONCLUSION

In this paper, we explored an innovative multi-objective optimization approach for the Green Vehicle Routing Problem (GVRP) by integrating vehicle speed as a fuzzy parameter within the framework of the Comprehensive Modal Emissions Model (CMEM) called Fuzzy-CMEM. This modeling approach allows for a more accurate representation of real uncertainties related to travel speeds on road segments, due to dynamic factors such as traffic or weather conditions. By incorporating these fuzzy speeds, it became possible to compute fuel consumption more realistically. The results demonstrate that this approach improves the relevance of optimization decisions by providing more robust solutions that better reflect real-world conditions and classical CMEM model severely underestimates fuel consumption compared to our Fuzzy-CMEM model. Indeed, the simultaneous consideration of minimizing travel distance and fuel consumption under uncertainty enables a balance between logistical performance and environmental impact. The FMOGVRP based on the Fuzzy-CMEM opens several promising avenues for future research, such as: Developing a fully fuzzy multi-objective optimization framework, in which Pareto dominance is handled directly in the fuzzy domain without relying on an initial defuzzification step. Integrating real-time or dynamic speed data into the proposed model, as this would enable adaptive routing strategies and improve the robustness of eco-logistics decisions under time-varying conditions. Finally, focusing on large-scale industrial applications and benchmark-based evaluations, as well as the development of specialized metaheuristics designed to handle both fuzziness and multi-objective optimization more efficiently.

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