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Analysis of the Impact of Lateral Stock Transfers in Distribution Network with a Central Warehouse and Two Storage Points

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ABSTRACT

Our study, Analysis of the impact of lateral Transfers in a stock distribution system with a central warehouse and two stocking points, aims to analyze the effet of lateral stock transfers between the two stocking points on minimizing the total inventory management cost system, retailers manage their inventories according to the (R,S) policy. This study also examines the service level and the stockout rate resulting from the implementation of lateral stock transfers. Each point i (i=1,2) manages its inventory independently in order to meet the consumer demand yi. Each stocking point has a maximun inventory level Si , when customer demand is less than or equal to the reorder point si , an order of quantity Qi= Si-si is placed with the central warehouse. This quantity is delivered after a known lead time Li. If the delivery lead time is too long, stocking point i may request a lateral transfer of quantity Xji from stocking point j, which has excess inventory, in order to avoid a stockout. The originality of this publication stems from the implementation of a numerical application using MATLAB, which allowed us to conduct this analysis.

I. Introduction

Since ancient times, inventory management has been a concern for humankind. Competition among companies has intensified considerably, compelling them to develop strategies to remain competitive, one of which is lateral stock transfer. The inventory distribution network under analysis consists of two levels and two storage locations: the first level comprises the two storage points, while the second level is occupied by the central warehouse [7].

Inventory can be defined as follows: Inventory is a collection of goods, raw materials, supplies, packaging, and waste that are stored in a warehouse for future use.[6]

We assume that the central warehouse can supply the storage points without a stock shortage because its capacity is unlimited. We conduct our study using the (R,S) management policy, where R is the order release period and S is the maximum stock or replenishment level specific to each site. Inventory management is the process of ordering, storing tracking, and controlling, a company's inventory to ensure the right quantity of stock is available at the right time, minimizing costs and meeting customer demand [11].

The objective of inventory management is to maximize the company's profit, minimize the total inventory costs, and enhance the customer service rate [3]. Service level is defined as the probability that demand is fulfilled; it comes into play when demand is random. Stock distribution flows from the second level (central warehouse) to the first level (the two storage points). This forms a stock distribution network.

Lateral stock transfer occurs when one storage point has a surplus of stock while another storage point is out of stock. In this case, stock is transfered from the storage point with the surplus to the one experiencing a stockout.

The purpose of the collaboration between the points is to pool stock in order to meet customers' random demand.

We assume that retailer demand is independent and identically distributed (i.i.d.). The methodology is based on the continuous review (R,S) inventory policy system; at the end of each period R, an order is issued to bring the stock back to the replenishment level S. We used the normal (Gaussian) distribution N(μ,σ), where μ, is the mean of the demand and σ, is the standard deviation of the demand. [15] Using probability density and the normal distribution function, we established the various mathematical expectations involved in the formula for the total cost of inventory management. We partially derived the total cost of inventory management and obtained two equations. Solving these equations using Matlab software allowed us to calculate the inventories that minimize the total cost. Finally, we analyzed the various inventory management parameters.

Lateral stock transfer becomes particularly beneficial when the lead time for delivering goods from the central warehouse to the storage points is significant, and the geography of the storage points must also be taken into account [9].

Our study includes two key points:

- Inventory management in a distribution network consisting of a central warehouse and two storage sites.
- Numerical application.

II. Literature Review

We present, in summary, some studies that have addressed multi-echelon and multi-site inventory management. Moalla Z., Banroum, And Tlill M., (2010) The authors address the problem of inventory management with lateral transhipment in a multi-echelon distribution network with multiple storage locations. They consider the inventory (R s S). Consumers place random demands at the storage points. They demonstrated the benefits in a product distribution network resulting from transhipments between storage points.

Anthony (2O17) the author analyzed the use of lateral stock transfers between an entreprise's storage points. He demonstrated the positive effect of lateral transfers on maximizing profit at each storage location.

Axsäter (2003) , he studied the theory of multi – echelon systems using the (R,Q) inventory model, where R each stocking point has a reorder level and Q is the fixed order quantity.

III. Inventory Management In A Distribution Network Comprising A Central Warehouse And Two Retail Outlets

The network under study consists of a central warehouse and two storage sites, which supply products to customers whose demands occur at site1 (D1) and site2 (D2). We assume that customer demand Di (i = 1, 2) follows a normal distribution with a given mean μi (i=1,2) and standard deviation σi (i= 1, 2).

Each storage site fulfils customer demand Di or yi (i=1,2). In the event of a shortage or threat of shortage, it can request a quantity Xij of goods from the other storage point. This is the lateral transfer between storage point i, which has a surplus of stock, and storage point j, which faces a shortage or threat of stockout. This transfer is carried out according to the rules concerning the lateral transfer cost, the storage cost, the ordering cost, and the stockout cost [1]. Lateral stock transfer will only be beneficial if the delivery time between storage points 1 and 2 is shorter than the delivery time between the central warehouse and the storage points; otherwise, an order is placed from the central warehouse. We also note that customers are only in contact with the storage sites and not with the central warehouse. We used the (Ri,Si) policy, the periodic review policy. The procedure for this policy is as follows: order a variable quantity of goods that can bring the stock position up to the replenishment level Si at each period Ri.

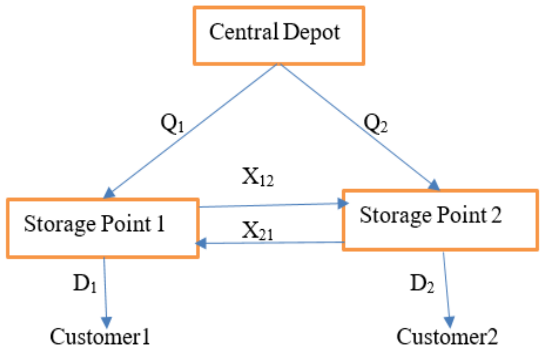

Figure 1. Two-level distribution network with a central warehouse and two storage locations.

IV. Mathematical Modelling Of The Problem Without Considering The Customer Satisfaction Rate At The Storage Points

A Problem description

The primary objective of the mathematical modeling of this problem is to determine the optimal inventory level S_i^* (i=1, 2) at both stocking points that minimizes the total inventory management cost.[17] The total inventory cost is the sum of various costs: ordering costs, transshipment or lateral transfer costs, storage costs, and stockout costs.

$$C_T(s) = \sum_{i=1}^{2} c_i E(Q_i) + \sum_{\substack{i=1 \\ i\neq j}}^{2} \sum_{j=1}^{2} c_{ij} E\left(X_{ij}\right) + \sum_{i=1}^{2} c_{si} E(I_i^+) + \sum_{i=1}^{2} c_{ri} E(I_i^-) \quad (1)$$

where:

- c_i $E(Q_i)$ is the average ordering cost, where $E(Q_i)$ is the average order quantity order quantity at storage point i, and c_i is the ordering cost per unit.
- $c_{ij}E(X_{ij})$ is the average lateral transfer cost from storage point i to storage point j, where $E(X_{ij})$ is the average

transferred quantity, and c_{ij} is the lateral transfer cost per unit.

- $C_{si}E(I_i^+)$ is the average holding cost at site i, where carrying $E(I_i^+)$ is the average inventory held at storage site i, and c_{si} is the holding cost per unit.
- $C_{ri}E(I_i^-)$ is the average stockout cost at storage point i, where $E(I_i^-)$ is the average stockout level at storage point i, and C_{ri} is the stockout cost per unit.

B Variables and Parameters Used

1. Parameters

- c_i is the holding cost per unit of goods at storage point i, per period, expressed in monetary units;
- c_{si} is the holding cost per unit, per period at site i (i=1,2), expressed in monetary units ;
- c_{ri} is the stockout cost per unit of goods at storage point i, expressed in monetary units per unit of goods, per period;
- c_{ij} denotes the unit cost of lateral transfer from storage location i to storage location j;
- $C_T(s)$ represents the total management cost par period, expressed in monetary terms;
- β_i^{av} is the customer service level at storage point, before the lateral stock transfer;
- β_i^{ap} is the customer service level at storage point i, after the lateral stock transfer ;
- P_i^{av} is the probability of no stockout at storage point i, before the lateral stock transfer ;
- P_i^{ap} is the probability of no stockout at storage point i, after the lateral stock transfer.

2. Variables

We denote by:

- -Qi is the quantity of items ordered by each storage point i from the central warehouse.
- I_i^+ denotes the available inventory at storage location i following the lateral stock transfer;
- I_i^- denotes the shortage at storage location i following the lateral stock transfer;
- I_i is the actual stock at storage point i, after cross-docking , is the difference between the available stock and the shortage stock ;
- X_{ij} is the quantity of lateral stock transfer from storage point i to storage point j that is experiencing a shortage;
- -y_i is the quantity demanded by the customer at storage location;
- $f(y_i)$ denotes the probability density function (PDF) of the normal distribution associated with the demand yi ;
- $F(y_i)$ denotes the cumulative distribution function (CDF) of the normal distribution associated with the demand yi.

C. Complete collaboration between the storage locations.

Complete collaboration is defined as follows:

1. If $\forall i$ $D_i \leq s_i$, $\forall j$, $D_j \leq s_j$, demand is less than or equal to available inventory, then $X_{ij} = X_{ji} = 0$, $\forall i,j = 1,2$, $i \neq j$, this shows that there is no lateral transfer of inventory.

2 If $\forall i$ $D_i < s_i$ and $\forall j$, $D_j > s_j$, the storage point j is in shortage or out of stock due to demand D_j while storage point i has excess inventory, the quantity transferred from site i to site j is:

$$X_{ij} = \min(s_i - D_i, D_j - s_j) \qquad \forall i,j = 1,2, i \neq j$$

Where: $D_i = y_i$ and $D_j = y_j$.

D. Cases where lateral transfer is prohibited

1 Lateral transfer is impossible if the two storage points are geographically distant ; in this case, the transshipment time l_i is longer than the delivery time L_I ($l_i > L_i$) .

2 If both storage units i and j each have a positive quantity of stock ($I_1 > 0$ and I_2), then lateral work between these two units is impossible $X_{12} = X_{21} = 0$.

3 If both storage units are out of stock or have insufficient quantities ($I_1 < O$ and $I_2 < 0$), then lateral work between these two units is impossible $X_{12} = X_{21} = 0$.

E. The mathematical expectation expressions used in this study.

1. The mathematical expectation of the lateral transfer between storage locations i and j.

$$E(X_{ij}) = \int_{y_i=0}^{s_i} F_i(y_i)[1 - F_j(s_i + s_j - y_i)]dy_i \qquad (2)$$

Where: s_i and s_j s are the actual inventories at storage points i and j.

2. The mathematical expectation of the avaible inventory at storage location i following the lateral stock transfer.

$$E(I_i^+) = \int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i \qquad (3)$$

3 The mathematical expectation of the shortage inventory at storage

location i following the lateral stock transfer.

$E(I_i^-) = E(y_i) - s_i + \int_{y_i=0}^{s_i} F_i(y_i)dy_i - \int_{y_j=0}^{s_j} F_j(y_j)dy_j + \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j(s_i + s_j - y_i)dy_i$ (4)

Where: $E(y_i) = \int_{y_i=0}^{\infty} y_i f_i(y_i)dy_i$
is the average of demand in the site of i storage.

4 The mathematical expectation of net stock.

$$E(I_i) = E(I_i^+) - E(I_i^-) \quad (5)$$

Replacing (3) and (4) in (5) we have:

$E(I_i) = s_i - E(y_i) - \int_{y_i=0}^{s_i} F_i(y_i)dy_i + \int_{y_j}^{s_j} F_j(y_j)d\,y_j - \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j\,(s_i + s_j - y_j)dy_i + \int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i$ (6)

The mathematical expectation of the quantity ordered Q_i by location i.
The quantity ordered $Q_i = s_i - I_i$, from this expression we obtain the expression for the average quantity ordered.

$E(Q_i) = E(y_i) + \int_{y_i=0}^{s_i} F_i(y_i)dy_i - \int_{y_j}^{s_j} F_j(y_j)d\,y_j + \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j\,(s_i + s_j - y_j)dy_i - \int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i$ (7)

F. Expressions of the different average cost.

1 Analytical expression of the ordering cost function.

$C(Q_i) = c_i(E(y_i) + \int_{y_i=0}^{s_i} F_i(y_i)dy_i - \int_{y_j}^{s_j} F_j(y_j)d\,y_j + \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j\,(s_i + s_j - y_j)dy_i - \int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i)$

2 Analytical expression of the average inventory holding cost

$C(I_i^+) = c_s(\int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)d\,y_i))$

3 Analytical expression of the average stockout cost.

$(C(I_i^-) = c_{si}\,(\,E(y_i) - s_i + \int_{y_i=0}^{s_i} F_i(y_i)dy_i - \int_{y_j=0}^{s_j} F_j(y_j)dy_j + \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j(s_i + s_j - y_i)dy_i)$

4 Analytical expression of the average lateral transfer cost

$$C(X_{ij}) = c_{ij}(\int_{y_i=0}^{s_i} F_i(y_i)[1 - F_j(s_i + s_j - y_i)]dy_i)$$

G.Total inventory Management Cost Expression.

By replacing (2), (3), (4), and (7) in (1) and considering two sites i and j, we obtain the expression of the total cost of management for the network under study.

$C_T(s) = (c_i + c_{ri})E(y_i) + (c_j + c_{rj})E(y_j) + (c_{ri} + c_{ij} + c_i - c_j - c_{rj})\int_{y_j=0}^{s_j} F_j(y_j)dy_j + (c_{rj} + c_{ji} + c_j - c_i - c_{ri})\int_{y_i=0}^{s_i} F_i(y_i)dy_i + (c_{si} - c_i - c_{ij})\int_{y_i=0}^{s_i} F_i(y_i)F_j(s_i + s_j - y_i)dy_i + (c_{sj} - c_j - c_{ji})\int_{y_j=0}^{s_j} F_j(y_j)F_j(s_j + s_i - y_j)dy_j + (c_{ri} + c_i)\int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j(s_j + s_i - y_i)dy_i + (c_{rj} + c_j)\int_{y_j=s_j}^{s_i+s_j} F_j(y_j)F_i(s_j + s_i - y_j)dy_j - c_{ri}s_i - c_{rj}s_j$ (8)

G.. Determination of stock levels s_i^* and s_j^* that minimize the total cost.

We will determine s_i^*, s_j^* minimizing total cost by partially differentiating (8) with respect to si and sj using Leibniz's rule of derivation.

$\frac{\partial C_T(s)}{\partial s_i} = (c_i - c_j + c_{ri} - c_{rj} + c_{ij})F_i(s_i) + (c_{si} - 2c_j - c_{ij} - c_{ri})F_i(s_i)F_j(s_j)\int_{y_i=0}^{s_i} F_i(y_i)f_j(s_i + s_j - y_i)dy_i + (c_{si} - c_i - c_{ij})\int_{y_j=0}^{s_j} F_j(y_j)f_i(s_i + s_j - y_j)dy_j + (c_i + c_{ri})\int_{y_i=s_i}^{s_i+s_j} F_i(y_i)f_j(s_i + s_j - y_i)dy_i + (c_j + c_{rj})\int_{y_j=s_j}^{s_i+s_j} F_j(y_j)f_i(s_i + s_j - y_j)dy_j - c_{ri} = 0$ (9)

$\frac{\partial C_T(s)}{\partial s_j} = (c_j - c_i + c_{rj} - c_{ri} + c_{ji})F_j(s_j) + (c_{sj} - 2c_j - c_{ji} - c_{rj})F_i(s_i)F_j(s_j)\int_{y_j=0}^{s_j} F_j(y_j)f_i(s_i + s_j - y_j)dy_j + (c_{sj} - c_j - c_{ji})\int_{y_i=0}^{s_j} F_i(y_i)f_j(s_i + s_j - y_i)dy_i + (c_j + c_{rj})\int_{y_j=s_j}^{s_i+s_j} F_j(y_j)f_i(s_i + s_j - y_j)dy_j + (c_i + c_{ri})\int_{y_i=s_i}^{s_i+s_j} F_i(y_i)f_j(s_i + s_j - y_i)dy_i - c_{rj} = 0$

Since the analytical resolution of this system (9) is difficult, we will use MATLAB's fsolve function to determine the stock levels that minimize the total inventory management cost. (See 5. Numerical Application)

H.Mean and standard deviation of demand when the lead time isnot negligible.

-Mean demand depending on the review period R and lead time L.

$$\mu_{L+R} = \mu(L+R)$$

-Standard deviation of demand as a function of the review period and the lead time L

$$\sigma_{L+R} = \sqrt{L + R}\,\sigma$$

I. Contribution of transshipment between storage points.

We will examine two possibilities necessary to detect the contribution of lateral transfer:
Transshipment without constraints and transshipment subject to customer satisfaction constraints.

1 Optimization of the customer service level without constraints.

The main goal of transshipment is to enhance customer service levels by avoiding stockouts at any storage location. The probability of stockouts not occurring and the fill rate or customer satisfaction.

- Probability of stockouts not occurring at storage point i after lateral transfer.

$$P_i^{ap} = P(y_i \leq s_i) + P(s_i < S_i < s_i + s_j - y_j, y_j < s_j)$$

where : $P(y_i \leq s_i) = \int_0^{s_i} f_i(y_i)dy_i = F_i(s_i)$ (10)

The probability of no stock shortage at storage point i before transshipment is:

$$P_i^{ap} = F_i(s_i) + \int_{y_j=0}^{s_j} \int_{y_i=s_i}^{s_i+s_j-y_j} f_i(y_i) f_j(y_j) dy_i dy_j \qquad (11)$$

The customer satisfaction rate after transshipment between site i and site j is expressed as follows:

$$\beta_i^{ap} = (\beta_i^{av} + E(X_{ji})) / E(D_i) \qquad (12)$$

where: $\beta_i^{av} = \left[\int_{y_i=0}^{s_i} y_i f_i(y_i) dy_i + \int_{y_i=s_i}^{\infty} s_i f_i(y_i) dy_i \right]$

After integration we have:

$$\beta_i^{av} = s_i - \int_{y_i=0}^{s_i} F_i(y_i) dy_i \qquad (13)$$

We have the expression for the filling rate before lateral transfer. Replacing (13), (2) in (12) and rearranging the terms, we have:

$$\beta_i^{ap} = 1 - (E(y_i) - s_i + \int_{y_i=0}^{s_i} F_i(y_i)dy_i - \int_{y_j=0}^{s_j} F_i(y_i)dy_i + \int_{y_i=s_i}^{s_i+s_j} F_i(y_i)F_j(S_i + - y_i)dy_i)/E(y_i)$$

Replacing the expression in parentheses with we have

$$\beta_i^{ap} = 1 - \frac{E(I_i^-)}{E(y_i)} \qquad (14)$$

This is the expression of the customer satisfaction rate after transshipment.

The stockout rate is given by:

$$T_r = \frac{E(I_i^-)}{E(y_i)}$$

I. Mathematical Modeling of a constrained inventory distribution network.

The mathematical model for constrained inventory management can be formulated as follows:

$$C_T(s) = \sum_{i=1}^{2} c_i E(Q_i) + \sum_{\substack{i=1 \\ i \neq j}}^{2} \sum_{j=1}^{2} c_{ij} E(X_{ij}) + \sum_{i=1}^{2} c_{si} E(I_i^+) + \sum_{i=1}^{2} c_{ri} E(I_i^-)$$

Subject to :

$\beta_i^{ap} \geq \beta_i$

With β_i=0,99, representing a 99% customer satisfaction rate.

Using (14), the constraint can be expressed as follows:

$\beta_i^{ap} \geq \beta_i \qquad E(I_i^-) = (1-\beta_i)E(y_i)$

Using substitution, the objective function is rewritten as:

$$C_T(s) = \sum_{i=1}^{2} c_i E(Q_i) + \sum_{\substack{i=1 \\ i \neq j}}^{2} \sum_{j=1}^{2} c_{ij} E(X_{ij}) + \sum_{i=1}^{2} c_{si} E(I_i^+) + \sum_{i=1}^{2} c_{ri} (1-\beta_i) E(y_i) \qquad (15)$$

By deriving (15) using Leibniz's formula, we obtain the following system of equations.

$$\frac{\partial C_T(s)}{\partial s_i} = (c_i + c_{ij})F_i(s_i) + (c_{si} - 2c_i - c_{ij})F_i(s_i)F_j(s_j) + (c_{si} - c_i - c_{ij}) \int_{y_i=0}^{s_i} F_i(y_i) f_j(s_i + s_j - y_i) d\, y_i + (c_i) \int_{y_i=s_i}^{s_i+s_j} F_i(y_i) f_j(s_i + s_j - y_i) d\, y_i = 0 \qquad (16)$$

$$\frac{\partial C_T(s)}{\partial s_j} = (c_j + c_{ji})F_j(s_j) + (c_{sj} - 2c_j - c_{ji})F_i(s_i)F_j(s_j) + (c_{sj} - c_j - c_{ji}) \int_{y_j=0}^{s_j} F_j(y_j) f_i(s_i + s_j - y_j) d\, y_j + (c_j) \int_{y_j=s_j}^{s_i+s_j} F_j(y_j) f_i(s_i + s_j - y_j) d\, y_j = 0$$

Given the complexity of obtaining an analytical solution to this system, we resort to MATLAB's fsolve function to determine the inventory levels that minimize the total management cost. (Refer to Section 5: Numerical Application).

V. NUMERICAL APPLICATION

In this study, we utilize Matlab's fsolve function to determine the optimal stock levels s_1^* and s_2^* that minimize the total inventory management cost. Additionality, we evaluate the expected values, the customer service level, and the probability of avoiding stockouts.
Input parameters

TABEL I
MEAN (μ) AND STANDARD DEVIATION (Σ) OF DEMAND

μ_1	μ_2	σ_1	σ_2
200	75	400	106

TABEL II
THE UNIT COSTS

Unit cost	Storage point1	Storage point2
c_{si}	1.1	1
c_i	22	20
c_{ri}	4	4
c_{ij}	3	3

A. Results after analysis

The table below presents the results obtained from running program using Matlab, which allowed us to evaluate the different parameters. Based on this table, we created additional tables to further analyze these parameters. The graphs were generated using Microsoft Excel.

TABEL III
OUTPUT RESULTS FROM THE PROGRAM EXECUTION FOR THE THREE MODELS

	Model without transfer		Model with unconstrained transfer		Model with constrained transfer	
i=1,2	Site1	Site2	Site1	Site2	Site1	Site2
s_i^*	310,9	624,0	226	411	271	494,4
C_{si}			644,31	1295,0	643,1	2166,7
$C_T(s)$	3785,9		1939,3		2809,8	
$E(I_i^+)$	47,18	55,48	72,51	119,35	119	198
$E(I_i^-)$	27,15	31,33	0,96	4,377	0,061	0,3965
$E(D_i)$			150,33	300,24	150,2	300,28
$E(X_{ij})$			7,1324	2,9181	8,076	0,1243
$E(Q_i)$			154,44	296,02	151,6	496,6
P_i^{av}			0,8788	0,8225	0,968	0,9497
P_i^{ap}			0,9623	0,9142	0,991	0,9805
β_i^{av}			0,9742	0,9617	0,995	0,9916
β_i^{ap}			0,9936	0,9854	0,985	0,9987

This table is obtained using MatLab software ; we have considered the lead time to be negligible.

1. Comparison of the total costs of the three models.

We will compare the total management cost for following systems: without transshipment, with unconstrained lateral transshipment, and with constrained lateral transshipment.

TABEL IV
TOTAL COST FOR EACH TYPE OF MANAGEMENT MODEL

	A	B	C
Total cost	3785,9	2038,1	2809,8

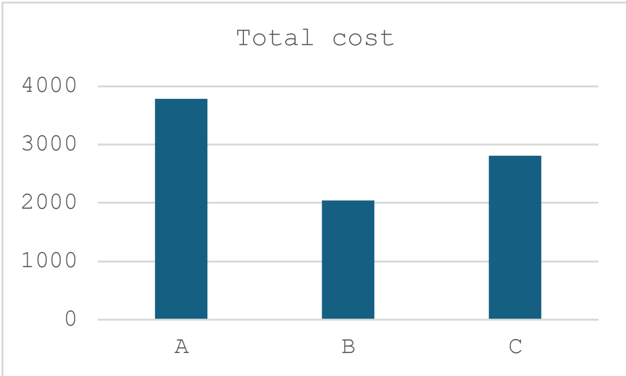

Figure 2: Comparison of the different total management costs for the three cases.
Where: A means a network without lateral transhipment.
B indicates a network tha uses unconstrained lateral transhipment.
C indicates a network where sites collaborate subject to constraints.

We observe that the total cost is higher in the inventory management model without lateral transshipment compared to the model in which sites are allowed to transfer their stock. The total cost in the case of constrained lateral transshipment is higher than in the unconstrained transshipment scenario. This difference is mainly due to the significant amount of inventory available in the constrained inventory management model. The implementation of lateral transfer leads to a 25,7% reduction in total cost

2. Analysis of customer satisfaction levels under two inventory strategies

TABEL V
CUSTOMER SATISFACTION RATES FOR THE TWO SCENARIOS OF LATERAL INVENTORY TRANSFER

	Lateral transfer without constraint		Lateral transfer under constraint	
	Site1	Site2	Site1	Site2
β_i^{av}	0,9742	0,9617	0,9947	0,9916
β_i^{ap}	0,9936	0,9854	0,9996	0,9987

β_i^{av}= βiav : Pre – transfer customer satisfaction rate.
β_i^{ap}= βiap: Post-transfer customer satisfaction.

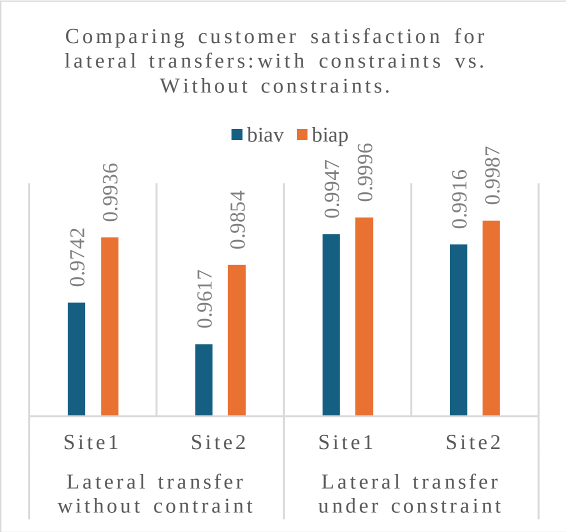

Figure 3: Comparison of customer satisfaction levels pre-and post – lateral stock transfer.

Customer satisfaction rates increased following lateral stock transfer in both constrained and unconstrained scenarios.

3. Study of the probability of non-stockout in the two stock transfer models with or without constraint.

TABEL VI
STOCKOUT PROBABILITY FOR THE TWO STOCK TRANSFER MODELS WITH OR WITHOUT CONSTRAINTS

	Unconstrained lateral transfer		Constrained lateral transfer	
	Site 1	Site 2	Site 1	Site 2
P_i^{av}	0,8788	0,8225	0,9687	0,9460
P_i^{ap}	0,9623	0,9142	0,9913	0,9997

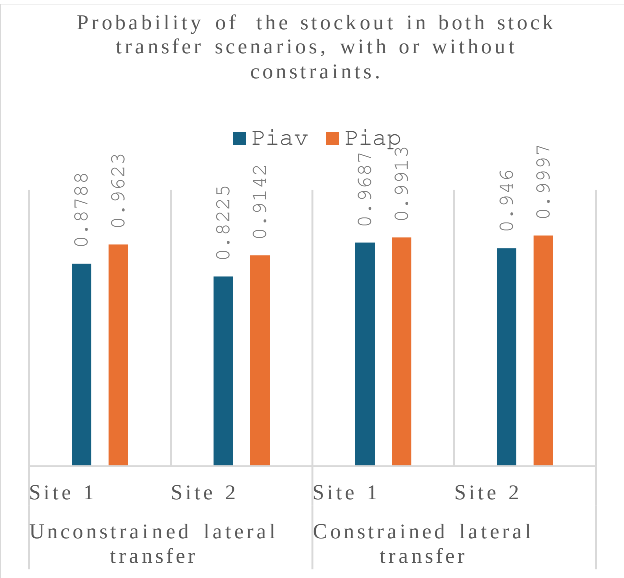

Figure 4: Comparison of non – failure probability in lateral transfer under constrained and unconstrained conditions.

where: Piav (P_i^{av}) represents the probability that no failure occurs before lateral transfer at inventory point i. and Piap (P_i^{ap}) is the probability of non – failure after lateral transfer at stock point i.

The stockout risk decreased at both locations (1 and 2) following the lateral stock transfer, indicating that such transfers help mitigate stockout risk. This reduction is notably more significant in the model involving constrained lateral transfers.

B. Effect of inventory variation on customer service level and post-transfer non-stockout probability at storage point 1. (Unconstrained transfer)

TABLE VII
VARIATION IN CUSTOMER SERVICE LEVEL AND NON – STOCKOUT PROBABILITY

S_i (i=1,2)	Customer satisfaction rate site 1	Probability of non – failure site 1
226-411	0,9261	0,7948
246-431	0,9468	0,8278
266-451	0,9625	0,8548
286-471	0,9742	0,8762

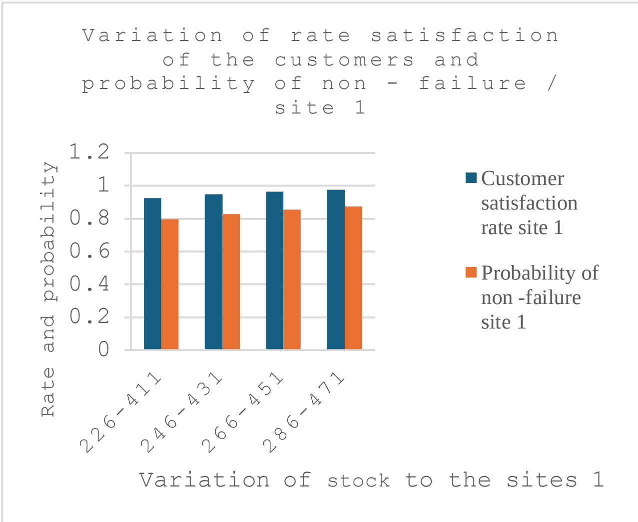

Figure 5: Variation in customer satisfaction rate and non – stockout probability at site 1 following stock level changes at sites 1.

We observe that the increase in available stock at the storage sites leads to an improvement in customer satisfaction rate and the probability of non – stockout at the site 1.

C. Variation in total cost as available stock varies at both sites.

TABEL VIII
MANAGEMENT COST VARIATION AS A FUNCTION OF STOCK AVAILABILITY AT STORAGE POINTS 1 AND 2

Inventory variation at sites 1, 2	Total cost
50-150	2085,5

100-200	2050,2
150-250	2036,5
200-300	2028
226-411	2038,1
250-440	2045,6
300-500	2068

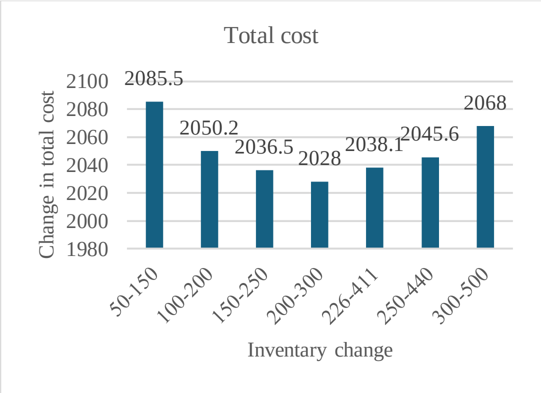

Figure 6: Variation in total inventory management cost based on stock availability changes at both sites in unconstrained collaboration.

By waring the available stock at the two collaborating sites, we observe that the total inventory management cost reaches its minimum when the stock level is 200 at site 1 and 300 at site 2.

D. Effect of demanded (standard deviation) on total inventory management cost.

TABEL IX
VARIATION IN DEMAND STANDARD DEVIATION AND TOTAL INVENTORY MANAGEMENT

Standard Deviation Site1 and 2	Total cost
75-100	1938,4
100 - 125	1957,4
150-175	2026,8
200-250	2153,4
300-350	2426,6

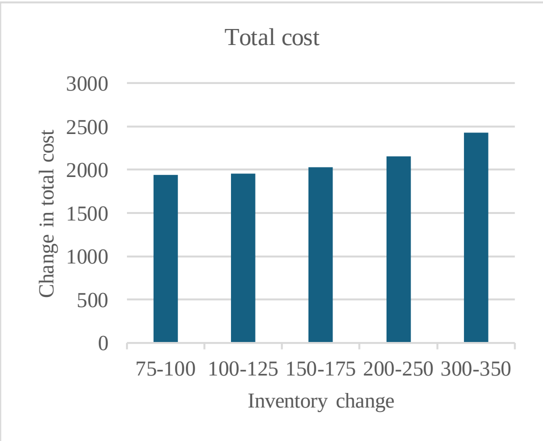

Figure 7: Changes in total inventory management cost with varying demand standard deviation.

It is observed that higher demand variability (standard deviation) results in increased total inventory management costs.

E. Comparison of the expected values of available stock and stockout levels across the three inventory management systems.

TABEL X
EXPECTED VALUES OF AVAILABLE STOCK AND STOCKOUTS IN THE THREE MODELS

	Non - stock transfer model		Unconstrained stock transfer model	Stock transfer model with constraints
I=1,2	Site1	Site2	Site1	Site2
EIi	47,189	55,48	72,51	198
EIri	27,15	31,33	0,96	0,3965

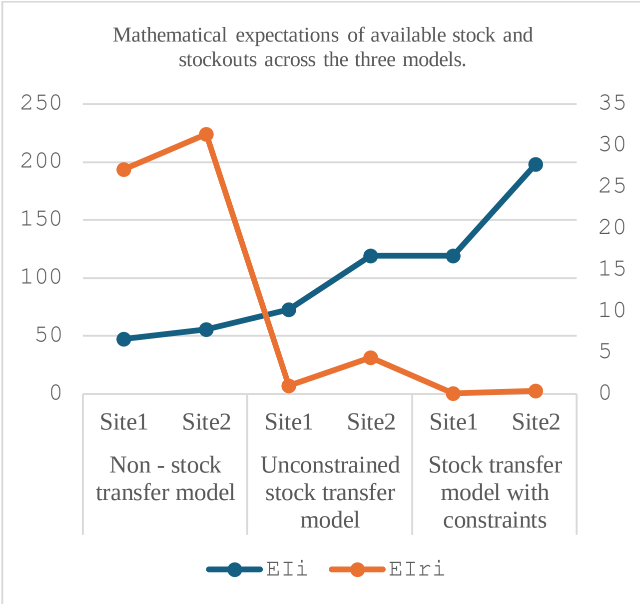

Figure 8: Expected values of available stock and stockouts in the three models.

We observe that the risk of stockout or shortage is very high in the stock management model without transfers. This explains why the expected value of stockouts is significantly higher for this model. In constrast, the model with constrained stock transfers presents a lower risk of shortage compared to the unconstrained transfer model. Similarly, the available stock tends to remain higher in transfer – based models, particularly in the case of constrained transfers.

F. Cases where the delivery time is not negligible

1 Effect of delivery time on overall inventory management cost.

TABEL X1
INFLUENCE OF LEAD TIME ON TOTAL STOCK MANAGEMENT COST

L1	L2	TOT. Cost
0	0	19695
1	1	37923
2	2	57635

Where: L1 is the delivery lead-time to site1 from the central depot. L2 is the delivery lead time to site2 from the central depot.

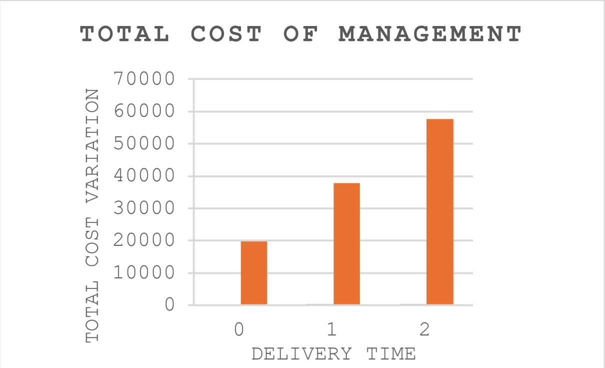

Figure 9: the effect of delivery lead time on total management cost.

We note that the total management cost increase as delivery lead time rises in inventory – sites, which may encourage market speculation.

2 Effect of delivery lead-time variation on the service level before and after lateral stock transfer

TABEL XII
EFFECT OF DELIVERY LEAD TIME VARIATION ON SERVICE LEVELS BEFORE AND AFTER LATERAL STOCK TRANSFERS.

L1=L2	T^{b}_{x1}	T^{b}_{x2}	T^{af}_{x1}	T^{af}_{x2}
0	0,9902	0,9958	0,9995	0,9991
1	0,859	0,7325	0,8659	0,7827
2	0,6489	0,4997	0,6493	0,5085

Where: T^{b}_{x1} is The service level of site1 before lateral transhipment under the influence of delivery lead time.
T^{b}_{x2} is The service level of site2 before lateral transhipment under the influence of delivery lead time.
T^{a}_{x1} The service level of site1 after lateral transhipment under the influence of delivery lead time.
T^{a}_{x2} The service level of site2 after lateral transhipment under the influence of delivery lead time.

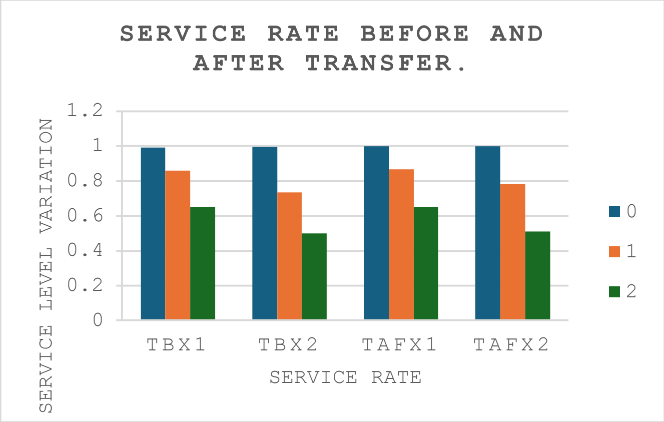

Figure 10: Influence of delivery lead-time on service level.

TBX1 is T^{b}_{x1}, TBX2 is T^{b}_{x2}, TAFX1 is T^{a}_{x1} and TAFX2 is T^{a}_{x2}

We note that the service level declines at both sites, before and after the lateral stock transfer, as a result of the increased delivery lead – time.

3. Stock-out probability and lead-time.

TABEL XIII
EFFECT OF DELIVERY LEAD TIME VARIATION ON STOCK – OUT PROBABILITY.

L1	L2	Pro.n.1 bf t	Pro.n.2 bf t	Pro.n.1 af.t	Pro.n.2af t
0	0	0,9772	0,9332	0,996	0,9702
1	1	0,5	0,144	0,5316	0,236
2	2	0,1241	0,0047	0,1247	0,0068

Where: Pro.n.1 bf t is the probability of no stockout at site 1 before lateral stock transfer.
Pro.n.2 bf. is the probability of no stockout at site 2 before lateral stock transfer.
Pro.n.1 af.t is the probability of no stockout at site 1 after lateral stock transfer.
Pro.n.2 af.t is the probability of no stockout at site 2 after lateral stock transfer.

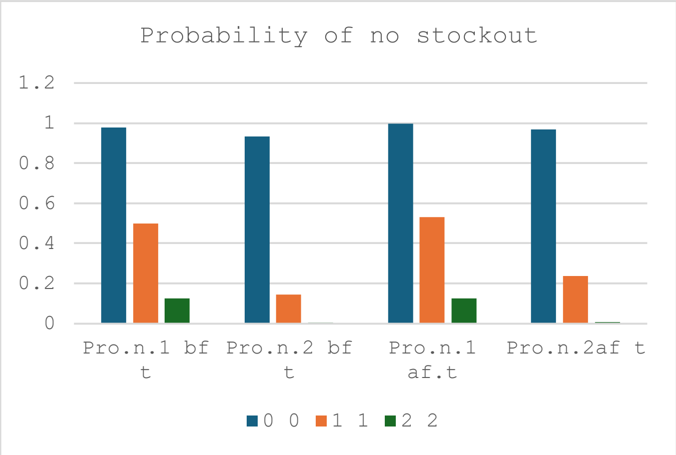

Figure 11: Effect of delivery lead-time variation on stock – out probability

The probability of no stockout decreases as the delivery lead-time increases, in both cases, before or after lateral transfer.

4 Influence of delivery lead time on the stockout rate.

TABEL XIV
EFFECT OF DELIVERY LEAD TIME ON THE STOCKOUT RATE

L1=L2	Sto.rate sit1	Sto. rate sit. 2
0	0,0047	0,0089
1	0,13	0,21
2	0,3509	0,4915

Where: sto.rate sit1is Stockout rate site1.
Sto.rate sit2 is Stockout rate site 2.

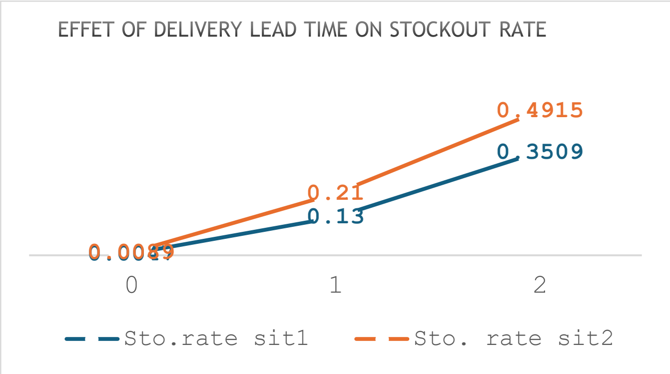

Figure 12: Influence of delivery lead-time on stockout rate.

We notice that the stockout rate at both sites increases with delivery lead-time.

VI. CONCLUSION

We conducted our study on the analysis of the impact of lateral stock transfers in a distribution network with a central warehouse and two storage points. The storage locations submit their replenishment requests to the central warehouse, which can fulfill the demands without facing stock shortages. The objective of our analysis was to examine the parameters necessary to minimize the total inventory management cost, to observe customer service level behavior, and to evaluate the probability of avoiding stockouts following the introduction of lateral stock transfers within the distribution network.

The results show that the risk of stockouts is low for storage points that implement lateral stock transfers, while it remains significant for those that do not use lateral transfers.

The increase in delivery lead time leads to an increase in total inventory management cost, an increase in stockout rate and consequently including a drop in the probability of non – stockout.

The service level also experiences decrease following the increase in delivery lead-time.

Our study will be more complete if we increase the number of echelons and distribution points for stock.

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